Fully Predictive Model of RCP in Plastic Pipes

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ABSTRACT

This work presents a 3D coupled solid-fluid model for predicting fast failures in pressurised plastic pipes. It is developed within a unified computational procedure where both solid pipe and pressurising media are discretised using the Finite Volume method. The coupling is achieved across the pipe-fluid interface (pipe bore) via special interpolation procedure. Cohesive zone model, which describes the local separation (fracture) process, is incorporated into the model allowing the prediction of the crack growth along the pipe. The model is qualitatively validated against the Full Scale (FS) experimental observations on gas pressurised MDPE (PE80) 250SDR11 pipe. Predicted pressure and crack histories, crack front shape and behaviour of the pipe during rapid crack propagation (RCP) in general, agree with experimental observations.

INTRODUCTION

Most buried pipelines are made from advanced, special-purpose polyethylene resins, which are very tough under low loading rates and at temperatures above the glass transition. However, these materials are susceptible to catastrophic failure by Rapid Crack Propagation (RCP) under extreme conditions. A brittle crack initiated by impact or otherwise may run along a pipeline at speeds of 100-300 m/s, for as long as a pressurising medium remains within the pipe. RCP can only be sustained above a critical pressure, which can be determined from a balance between the crack driving force $G$ exerted by the loaded structure and the crack resistance ($G_{cr}$) of its material. Because the latter depends on the loading rates (and therefore the crack speed) and temperature, so does the critical pressure. For gas pipes, the crack driving force is developed by a complex interaction between the expanding-exhausting gas, the deforming/fracturing pipe wall and the surrounding ‘backfill’ of gravel or sand (in case of buried pipes). Its calculation is, arguably, the most complex problem of its kind in fracture mechanics.

A number of research groups have attempted to model this problem numerically. In particular, Southwest Research (1) developed a procedure using shell Finite Element method for the pipe and a Finite Volume (FV) method for the fluid. This approach is rather cumbersome and inefficient as it involves coupling of two different numerical schemes, and its applicability is restricted to thin-wall pipes due to shell assumptions. Further, this procedure cannot predict neither crack shape nor crack history. On the other hand, the Imperial College group has based its development on 3D FV method for both pipe and its content. In initial simulations (2-4), the effects of the fluid on the fracturing pipe wall were approximated by uniform pressure forces ahead of the crack front, linearly decaying to zero over a fixed distance $L$. 
behind it, in accordance with experimental observations (5-7). Crack driving force was calculated for various combinations of decompression lengths and crack speeds until $G = G_D$ criterion was satisfied. Computed critical pressure compared well with measured data from the standard Small Scale Steady State (S4) test (Fig. 1).

![Figure 1: Predicting critical pressure in S4 test](image1)

In the intermediate step in the development of a true computational solid-fluid interaction model, the fluid was represented using a 1D gas flow solution (8). It implicitly combined the previous finite-volume stress-analysis code, and a 1D compressible flow analysis describing gas discharge through the crack opening. The model predicted gas decompression profiles which were in very good agreement with experimental measurements (Fig. 2). The model was more efficient than the previous one, where the decay length of each pressure profile had to be prescribed, because it eliminated the need to simulate ‘redundant’ combinations of crack speed and decay length (4). In both procedures, however, the crack front was assumed to be straight through the pipe wall and the crack velocity was prescribed. The conventional holding back force concept was employed to simulate the crack advance.

![Figure 2: Profiles of pressure decay from an S4 test](image2)
A coupling scheme based on separate solution domains for solid and fluid was developed within single FV framework. It uses a common data structure, for both domains. This has several advantages over the use of different methods (e.g. FV for the fluid and FE for the solid). It enables internal transfer of information across the solid-fluid interface, avoiding the development, running and maintenance of an additional interface program. It is therefore much more efficient and economical. Furthermore, the implicit coupling which is more accurate than the explicit one, and which requires iterations to converge the solid and fluid domains within each time step, is only possible within a single numerical solution procedure.

Here, the fluid and solid parts of the solution domain have separate meshes, but there is a common interface between them. The solid and fluid models were combined within a single code FOAM [Field Operation And Manipulation, (9)] to model the transient behaviour of a flexible pipe containing a compressible fluid. The systems of equations were solved for each mesh, and interface conditions were exchanged: tractions from fluid to solid and velocities from solid to the fluid (Fig. 3). Both meshes remain fixed during the calculations. Small-strain analysis was performed for the pipe and mesh distortions were neglected. Eulerian frame of reference was used for the fluid, as conventional in computational fluid dynamics, and the information about the motion of the neighbouring pipe wall is passed to the fluid boundary via pipe bore velocities (10). The scheme uses two sets of ‘inner’ iterations; one for each mesh, and implicit coupling in time was achieved through a series of ‘outer’ iterations, which solves the total system to convergence within each time step (11). Alternatively, explicit coupling can be employed, which eliminates the outer iterative loop (Fig. 3). In this method convergence is not achieved within each time step and therefore the solution is less accurate, but it is much more efficient.

START OF TIME STEP

Solve fluid
- Continuity equation ($\rho$)
- Momentum equation ($\mathbf{V}$)
- Temperature equation ($T$)
- Equation of state ($\rho$)
- Pressure equation ($p$)
- Velocity correction ($\mathbf{V}$)

Solve structure
- Transfer traction from the fluid to the solid boundary on interface
- Momentum equation ($\mathbf{U}$)

Equate velocity at interface ($\mathbf{V}$) to solid wall velocity ($\partial \mathbf{U}/\partial t$)

END OF TIME STEP

(implicit scheme only)

Figure 3: Solution schemes for solid-fluid interaction.
PIPE RCP MODEL

There are two main issues which need to be addressed with special care in order to be able to predict RCP in pressurised pipes:

i) Predicting the crack propagation, i.e. the shape of the crack front and the crack history,

ii) Passing the information to the fluid domain about the transient crack propagation, which creates a gap through which the fluid can escape.

Both issues are illustrated in Fig. 4.

The first problem is solved by employing a local separation law to describe the fracture process. The holding back force concept used in the previous procedures (4, 8) was abandoned due to the lack of its predictive capability. Instead, Cohesive Zone Model (CZM) was used (12, 13). Here, the local separation process is prescribed in terms of a traction-separation law (also called cohesion-decohesion law). Crack initiation and subsequent growth can be determined directly in terms of CZM parameters: the strength of cohesion, critical separation distances and the area $G_D$ under the cohesion-decohesion curve (Fig. 4.a).

Parameters for the proposed local separation criterion can be defined from micro-mechanical models of the materials under consideration, or via direct measurements, or by fitting simulations to fracture test results. In the present work, a simple Dugdale model was employed, with prescribed constant cohesive stress $t_c$ and $G_D$, giving the critical crack opening displacement $\delta_c$ (Fig. 4.a). Although, in general, the knowledge of the crack path is not required, a straight axial path was assumed here for simplicity. However, no assumption was made regarding the crack front shape, which is, together with the craze extension in front of the crack, natural outcome of the analysis.

As regard the second issue, difficulties were experienced in coupling the fracturing pipe with the contained fluid. As the crack propagates and the pipe opens up, a special interpolation procedure was developed to pass this information across the interface to the fluid. This is because the crack-gap appeared creating the escape route for the fluid, which was no longer fully contained within the pipe (Fig. 4.b). The newly developed model is capable of simulating flow of incompressible or compressible fluid through the crack opening, which is potentially smaller than the discrete boundary representation (cell face). It was coupled with fracturing-pipe model by exchanging the pressure, velocities and crack geometry between fluid and pipe domains. The crack geometry provides the escape route for the fluid, thus defining the flow field, while the work of pressure forces exerted by the fluid on the fracturing pipe provides most of the crack driving force. Velocities of the pipe bore provide the information of the pipe wall motion to the contained part of the fluid. In order to accurately capture the geometry of the crack and its influence on the flow field irrespective of the resolution of the solid-fluid interface (and without following the mesh lines of the initial surface), three possible modes of interaction between fluid surface and fracturing pipe were considered:

1) Fluid cell-face fully covered with pipe,
2) Fluid cell-face fully uncovered, and
3) Fluid cell-face partly covered.

Coupling of the first two modes was straightforward. The third one was treated as a combination of the covered and uncovered part, each providing an appropriate contribution to the cell balance through a proportion of fixed-value (for covered part) and fixed-gradient
(uncovered part) boundary conditions. This proportion was determined by calculating the (un)covered fraction of the cell area (Fig. 4.b). On the other hand, passing the pressure values from the fluid to the pipe bore was reasonably straightforward as all solid cell-faces on the interface were always fully covered by the fluid, and standard pressure interpolation suffices.

Figure 4: Solid and fluid solution domains: a) crack propagation model; b) coupling.
MODEL VALIDATION

The newly developed solid-fluid-RCP interaction model was validated qualitatively by simulating a gas pressurised, 3m long 250SDR11 PE80 pipe. Detailed comparison of model predictions against either FS or S4 test results was not considered at this stage. It was rather decided to perform a qualitative test of the overall behaviour of this complex coupled system. For simplicity, the boundary conditions were chosen such that they would represent the FS test. Effects of backfill were not considered in the simulation.

The solid domain was discretised with 67968 cells (8 through the wall thickness, 36 circumferentially and 236 axially), while 77408 cells were used to represent the fluid domain (328 in the x-y base by 236 axially). The following boundary conditions were employed (see Fig. 4):
Solid domain: 1s, 2s and 5s – symmetry planes, 3s – cohesive zone model, 4s – traction free, 6s – prescribed x-displacement rate of 2.2 m/s (corresponding to chisel loading at 10 m/s speed) with z and y-tractions zero, followed by all tractions free for crack opening displacement > 25 mm.
Fluid domain: 1f, 2f – symmetry plane, 3f – fixed pressure and zero velocity gradient.

The initial nominal pressure was set at \( p = 5 \) bars (absolute).

The following materials’ properties, corresponding to 298 K, were used:
Solid (linear elastic Hookean sold): Young’s modulus \( E = 2.5 \) GPa, Poisson’s ration \( \nu = 0.4 \), Mass density \( \rho = 940 \) kg/m\(^3\).
Fluid (ideal gas): Dynamic viscosity \( \eta = 18.45 \) µPas, Specific heat \( C_v = 717.86 \) J/kgK, Gas constant \( R = 287.14 \) J/kgK, Density \( \rho = p/RT \) (T absolute temperature).

As for the cohesive zone model parameters, the maximum stress \( t_c = 20 \) MPa and fracture resistance \( G_D = 5 \) kJ/m\(^2\) were chosen, giving the critical crack opening displacement \( \delta_c = 0.25 \) mm.

The computations were performed using a constant time step of 5 µs for both domains, and the solution was run for 0.01 s.

Figure 5 shows the pipe-gas system at three different time instants: a) at time \( t = 3 \) ms, when the crack propagated about 0.8 m along the pipe, b) at \( t = 6 \) ms, when the crack length was about 1.65 m, and c) at \( t = 9 \) ms, with 2.4 m of the pipe being split-open by the crack. The corresponding pressure profiles at three locations on the pipe bore A, B and C (see Fig. 4) are shown in Fig. 6. The crack history of the crack front point at the pipe outer surface (G) (see Fig. 4), is presented in Fig. 7.
Figure 5: Predicted RCP along gas pressurised pipe at times: a) 3 ms, b) 6 ms, c) 9 ms.
Predicted results demonstrate remarkable resemblance with experimental observations from FS tests (14). The curved shape of the crack front leading at the bore and lagging behind at the pipe outer surface is well captured by the model, together with the pipe dimpling in the crack tip region (Fig. 5). Similar observations are reported for S4 tests as well (5-7). The model predicted pipe failure at an average speed of 262 m/s for a given nominal pressure of 5 bars. RCPs were usually reported at speeds between 100 and 200 m/s, in both FS and S4 tests, and the numerical prediction overestimates the experimental observations. It should be noticed that this is largely due to the maximum cohesive traction of 20 MPa, which was somewhat arbitrarily chosen. Further investigation is required to obtain an appropriate traction-separation curve. Also shown in Fig. 5 are the evolution of the craze region at the crack front, gas discharge from the crack gap which resembles choked (sonic) flow for much of the crack growth as expected for the high nominal pressure and small crack opening, and the pressure distribution.

Figure 6 gives a closer look at the pressure distribution along the crack-path line (A), and those in middle and at the bottom of the pipe (B and C respectively). Pressure is shown to drop in front of the crack due to the decompression wave generated by the gas discharge through the crack, and it rapidly decays behind the crack tip due to this discharge. It can also be noticed that the pressure distribution does not vary appreciably along the pipe circumference, which has been observed experimentally and is often the assumption in analytical solutions (8). Some oscillations of the crack speed can be seen along the pipe (Fig. 7), and this correlates with experimental observations from both FS (14) and S4 tests (15).
CONCLUSIONS

The powerful numerical tool was developed for predicting detailed behaviour of the coupled pipe-fluid-RCP system. The model predictions in terms of fluid flow and pressure distributions, pipe deformation and fracturing were validated qualitatively against FS test observations. Thus, the complex coupling procedure as well as the failure model were verified.

Given the appropriate material data and testing conditions, the model can be used for accurate, efficient and economical calculation of critical pressure, for both FS and S4 tests. It is believed that the model will not only supplement these tests by giving a full insight into the problem, but it will to a large extent reduce the number of expensive tests required to establish the critical pressure. It will in particular be useful for large diameter pipes for which experimental S4 rigs are not available, and FS test not affordable. The final goal is to develop the model for predicting brittle-tough transition of plastic pipes, and for this to be achieved further work on traction-separation law is required. It is believed that the combination of this material model and the new numerical methods described here offer great promise for future research on understanding and predicting brittle-tough transitions in plastic pipes.

REFERENCES