CFD Modelling of Elastohydrodynamic Lubrication

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Abstract

Usually elastohydrodynamic lubrication (EHL) is modelled using the Reynolds equation for the fluid flow and the elastic deformation is calculated following the Hertzian contact theory.

In this thesis a CFD approach for modelling EHL is established. The full Navier-Stokes equations are used which enables the entire flow domain to be modelled and which can resolve all gradients inside the contact. Liquid properties are introduced where the viscosity is piezo-viscous, shear-thinning and temperature dependent and where the density is a function of pressure. The phenomenon of cavitation is taken into account by two homogeneous equilibrium cavitation models which are compared with each other. For one cavitation model an energy equation is developed which considers the effects of heat conduction and convection, viscous heating and the heat of evaporation. The Hertzian contact theory is implemented and parallelised within the CFD method and validated against analytical solutions. Then, the cavitation models and the Hertzian contact theory are coupled together in a forward iterative manner.

The developed method is applied to glass-on-steel and metal-on-metal line contacts and isothermal results are compared to the Reynolds theory. Very good agreement was found with the Reynolds theory in most cases. For high viscosity, high velocity and rolling conditions small differences to the Reynolds theory were found. The influence of temperature is studied for a series of test cases and the results are compared to their isothermal counterparts. All thermal calculations under sliding conditions developed a temperature-induced shear-band which is closer towards the slower, thus hotter, surface. The thermal, high viscosity calculations under sliding conditions showed significant pressure variation across the film thickness due to very large viscosity gradients. The impact of temperature on the friction force is very significant.

Results of a three-dimensional, isothermal point contact are shown to demonstrate the feasibility of such calculations.

The developed method is capable of giving new insights into the physics of elastohydrodynamic lubrication, especially in cases where the usual assumptions of the Reynolds theory break down.
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Nomenclature

Greek Symbols

- $\alpha$: Vapour fraction
- $\alpha_p$: Pressure viscosity index $Pa \cdot s$
- $\alpha_{T,s}$: Thermal diffusivity of solid $W/(m \cdot K)$
- $\beta$: Thermoviscous constant $1/K$
- $\Delta t$: Time step $s$
- $\dot{\gamma}$: Shear rate $1/s$
- $\epsilon_{rel}$: Relative deflection error
- $\eta$: Piezo-viscous, shear-thinning and thermal viscosity $Pa \cdot s$
- $\eta_0$: Base viscosity $Pa \cdot s$
- $\eta_{Barus}$: Barus viscosity $Pa \cdot s$
- $\eta_{Eyring}$: Eyring viscosity $Pa \cdot s$
- $\eta_{Houpert}$: Houpert viscosity $Pa \cdot s$
- $\eta_{Reynolds}$: Reynolds viscosity $Pa \cdot s$
- $\eta_{Roelands}$: Roelands viscosity $Pa \cdot s$
- $\Gamma_{\phi}$: Diffusivity
- $\mu$: Dynamic viscosity of mixture $Pa \cdot s$
- $\mu_F$: Friction coefficient
\( \mu_l \) Dynamic viscosity of liquid \( \text{Pa}\text{s} \)

\( \mu_v \) Dynamic viscosity of vapour \( \text{Pa}\text{s} \)

\( \nu \) Kinematic viscosity \( m^2/s \)

\( \Phi \) Temperature \( K \)

\( \phi \) Flow quantity

\( \psi \) Mixture compressibility \( \text{s}^2/m^2 \)

\( \psi_l \) Compressibility of liquid \( \text{s}^2/m^2 \)

\( \psi_v \) Compressibility of vapour \( \text{s}^2/m^2 \)

\( \psi_{l,dow} \) Compressibility of Dowson liquid \( \text{s}^2/m^2 \)

\( \rho \) Density \( \text{kg/m}^3 \)

\( \rho_s \) Density solid \( \text{kg/m}^3 \)

\( \rho_{l,0} \) Liquid density at ambient pressure \( \text{kg/m}^3 \)

\( \rho_{l,dow} \) Dowson liquid density \( \text{kg/m}^3 \)

\( \rho_{l,sat} \) Liquid density at \( p_{vapour} \) \( \text{kg/m}^3 \)

\( \rho_{v,sat} \) Vapour density at \( p_{vapour} \) \( \text{kg/m}^3 \)

\( \sigma \) Poisson’s ratio

\( \sigma_s \) Surface tension \( \text{N/m} \)

\( \tau \) Viscous stress \( \text{Pa} \)

\( \tau_0 \) Eyring stress \( \text{Pa} \)

\( Pe \) Peclet number

**Roman Symbols**

A Diagonal matrix coefficients

b Source vector
\( \mathbf{d} \) Length Vector between two neighbouring cell centres \( m \)

\( H(\mathbf{u}) \) Off-diagonal matrix coefficients multiplied by their corresponding velocities

\( \mathbf{M} \) Square matrix

\( \mathbf{S} \) Surface area vector \( m^2 \)

\( \mathbf{u} \) Velocity \( m/s \)

\( \dot{\mathbf{u}} \) Velocity predictor \( m/s \)

\( a \) Sonic velocity \( m/s \)

\( a_c \) Contact radius \( m \)

\( a_s \) Sonic velocity of solid \( m/s \)

\( a_{t,dow} \) Sonic velocity of Dowson liquid \( m/s \)

\( C_{p,l} \) Heat capacity of liquid \( J/(kg \, K) \)

\( C_{p,v} \) Heat capacity of vapour \( J/(kg \, K) \)

\( C_p \) Heat capacity \( J/(kg \, K) \)

\( C_{v,s} \) Heat capacity of solid \( J/(kg \, K) \)

\( Ca \) Capillary Number

\( Co \) Courant Number

\( E_r \) Reduced Young’s modulus \( Pa \)

\( F_f \) Friction force \( N/m \)

\( G \) Elastic shear modulus \( Pa \)

\( G^* \) Dimensionless materials group

\( h \) Film thickness \( m \)

\( h_l \) Specific enthalpy of liquid \( J/kg \)

\( h_v \) Specific enthalpy of vapour \( J/kg \)
\( h_{0,l} \) Specific reference enthalpy of liquid \( J/kg \)
\( h_{0,v} \) Specific reference enthalpy of vapour \( J/kg \)
\( h_{\text{cl},0} \) Undeformed gap between solids \( m \)
\( h_{\text{evap}} \) Heat of evaporation \( kJ/kg \)
\( h_u \) Undeformed geometry \( m \)
\( k \) Thermal conductivity \( W/(mK) \)
\( k_l \) Thermal conductivity of liquid \( W/(mK) \)
\( k_s \) Thermal conductivity of solid \( W/(mK) \)
\( k_v \) Thermal conductivity of vapour \( W/(mK) \)
\( L \) Load \( N/m \)
\( p \) Pressure \( Pa \)
\( p_0 \) Ambient pressure \( Pa \)
\( p_{r,0} \) Roelands reference pressure \( Pa \)
\( p_{\text{vapour}} \) Vapour pressure \( Pa \)
\( q_f \) Heat flux from fluid to solid \( W \)
\( R \) Radius \( m \)
\( R_T \) Relaxation factor for temperature
\( R_{\text{def}} \) Relaxation factor for deflection
\( r_{j,\text{def}} \) Position of mesh point in a deformed mesh \( m \)
\( Re \) Reynolds Number
\( S_0 \) Combined constant of thermovisous constant and viscosity
\( S_\phi \) Source term
\( S_{i,j} \) Influence coefficient of temperature distribution of a stationary surface
$SRR$  Slide-to-roll ratio

$T$  Temperature

$t$  Time

$T_0$  Ambient temperature

$T_{cars}$  Carslaw-Jaeger temperature

$t_{cd}$  Characteristic deflection time

$u$  Velocity along $x$ axis

$U^*$  Dimensionless film thickness

$u_s$  Velocity of bounding solid $x$

$u_{ent}$  Entrainment velocity

$V$  Volume

$v$  Velocity along $y$ axis

$w$  Deflection

$W^*$  Dimensionless load

$We$  Weber Number

$x$  Spatial coordinate along the direction of flow

$y$  Spatial coordinate perpendicular to $x$ and film-thickness $h$

$Z$  Combined constant of pressure viscosity index and viscosity
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6.1 Finite volume deflection
Chapter 1

Introduction

Elastohydrodynamic lubrication (EHL) describes the lubricant film formation between two non-conforming, elastic machine elements which are loaded against each other and are in relative motion. Typical examples with high elastic modulus are contacts in ball and roller bearings, cams on tappets and gear teeth loaded against each other. The EHL theory is also applicable in contacts with low elastic modulus, which may be called soft-EHL, such as rubber seals and car tyres on a wet road.

Figure 1.1 shows an optical interference of the contact of a steel ball on a glass plate, representing an EHL contact with high elastic modulus [1]. The lubricant is entrained from the left, passing through the contact and leaving the domain on the right. The colour grading indicates the film thickness, showing a plateau in the centre and a horseshoe shaped constriction at right. On the right hand side of the figure the effects of fluid cavitation can be observed. Downstream of the contact there are no clear structures visible in the cavitating region. Below and above that region lubricant streamers are formed pouring into the cavitating region.

The maximum pressure encountered in realistic contacts is up to 4 GPa [2]. In such conditions the fluid properties are not Newtonian. The viscosity and density become pressure dependent. Shear-thinning occurs if the surface slide against each other sufficiently and a significant amount of heat can be produced by viscous heating which has also an impact on the viscosity.

Usually the fluid flow in EHL is modelled using a variation of the Reynolds equation [2], which is an integrated version of the Navier-Stokes equations across the film thickness. The elastic deformation is calculated following the Hertzian contact theory described in detail by Johnson [3]. This approach has been applied with great success and compares very well with experiments in realistic conditions.
However, there are some inherent limitations in the Reynolds approach. The pressure is assumed to be constant across the film thickness and gradients of fluid properties and velocity across the film thickness are either assumed to be zero or greatly simplified. These gradients are never truly resolved.

On the other hand, with a computational fluid dynamics (CFD) approach to EHL it is possible resolve all gradients across the film. The Reynolds approach is limited to the near-parallel contact region, whereas a CFD approach can model the entire fluid system. The cavitation treatment for the Reynolds equation is very simple whereas a CFD approach can deliver more sophisticated cavitation models which could, in principle, also include surface tension effects. Another advantage is the ease of implementation of complex fluid properties. In CFD, any kind of dependency on pressure, temperature, shear-rate or any other variable can be easily implemented and its effect studied.

Schäfer [4] and Almqvist [5] [6] [7] have employed a CFD approach to EHL using the commercial code CFX. They employed a forward-iterative coupling between flow and deflection and reached up to 0.7 GPa maximum pressure for a 2D line contact. They found good agreement with the Reynolds theory. They modelled cavitation by having density as a function of pressure of the form originally implemented by Dowson and Higginson, see Hamrock [8] for details. They noticed singularities in the solution of the momentum equation if shear-thinning is not in-
troduced and the pressure exceeds 0.7 GPa \cite{7}. The temperature equation was also solved and they made the viscosity pressure and temperature dependent \cite{6}, but they did not discuss temperature effects.

The aim of this work is to

- establish a CFD approach with appropriate cavitation treatment for two and three-dimensional EHL using the freely-available package OpenFOAM,

- apply the developed method to a series of test cases,

- compare the results to the Reynolds theory,

- study the effect of temperature in detail and

- give recommendations for future development.

1.1 Thesis Layout

In chapter 2 the Reynolds theory is introduced and an overview of current modelling approaches is given. In Chapter 3 fluid modelling using the finite volume method (FVM) is described and two cavitation models are introduced and compared. In chapter 4 the implementation of the Hertzian contact theory of elastic bodies within the CFD framework is described. In chapter 5 the coupling of fluid modelling in chapter 3 and elastic deflection in chapter 4 is described and results are presented. In two dimensions the CFD results are compared to the Reynolds theory and thermal effects are studied. The feasibility of three-dimensional calculations is demonstrated by showing results of a 3D case. In chapter 6 recommendations for future development are made. In chapter 7 this work is summarised and conclusions are drawn.
Chapter 2

Reynolds Based EHL Models

2.1 Introduction

In this chapter the traditional Reynolds theory for elastohydrodynamic lubrication is introduced. The governing equations of the fluid, the fluid properties and modelling of the deforming solids are described. An overview of numerical techniques and recent advances is given.

2.2 Governing Equations

2.2.1 Reynolds Equation

The Reynolds equation which describes the pressure distribution in a lubricant film was first obtained by Reynolds in 1886 \[9\]. It can also be derived by simplifying the Navier-Stokes equations with the assumptions listed below. \[2\]

1. Body forces are negligible.

2. Pressure is constant through the lubricant film.

3. No slip at the boundary surfaces.

4. The lubricant flow is laminar (low Reynolds number).

5. Inertia and surface tension forces are negligible compared with viscous forces.

6. Shear stress and velocity gradients are only significant across the lubricant film.
7. The lubricant is Newtonian.

8. The lubricant viscosity is constant across the film.

9. The lubricant boundary surfaces are parallel or at a small angle with respect to each other.

With these assumptions we arrive at the full Reynolds equation in two dimensions \[2\].

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{\eta} \frac{\partial p}{\partial y} \right) = \\
6 \left\{ \frac{\partial}{\partial x} [\rho h (u_1 + u_2)] + \frac{\partial}{\partial y} [\rho h (v_1 + v_2)] + 2 \frac{\partial}{\partial t} (\rho h) \right\}
\]

(2.1)

Here \(x\) is the spatial coordinate along the direction of flow and \(y\) is the spatial coordinate perpendicular to \(x\) and the film-thickness \(h\), \(u_1\), \(u_2\), \(v_1\), \(v_2\) are the boundary surface velocities in \(x\) and \(y\) directions, \(\rho\) is the density, \(p\) is the pressure in the fluid film, \(\eta\) is the dynamic viscosity and \(t\) is time. This equation was originally developed for Newtonian flow, but can be adapted for more realistic fluids by making the viscosity a function of pressure, temperature, shear-rate and time.

Equation 2.1 can be simplified for one-dimensional, steady flow with \(u = (u_1 + u_2)/2\), neglecting time derivatives, to

\[
\frac{\partial}{\partial x} \left[ \frac{\rho h^3}{\eta} \frac{\partial p}{\partial x} \right] = 12 \frac{\partial}{\partial x} [\rho hu]
\]

(2.2)

The Reynolds equation can be adapted to realistic non-Newtonian behaviour including dependencies of viscosity on pressure and shear-rate and dependencies of density on pressure and temperature \[10, 11, 12, 13\]. Dowson introduced a generalised Reynolds equation which allows the variation of fluid properties across the film and he solved the energy equation for the fluid \[14\]. Spikes introduced an extended Reynolds equations which allows for the fluid to slip at the surfaces \[15\].

### 2.2.2 Geometry Equation

Due to high pressures in the contact region, the surfaces deform. The film thickness \(h\) between the two bodies describes the fluid film geometry of the problem, see figure
The deflection from the original shape to the deformed shape is assumed to be elastic. Often, the shape of a undeformed sphere in two dimensions or of a undeformed cylinder in one dimension is approximated by a parabola. This enables an analytical solution of the Reynolds equation. With $h_{c0}$ as the undeformed film thickness at the origin of coordinates, $h_u$ as the undeformed geometry and $w$ as the elastic deflection, as shown in figure 2.1, we obtain:

$$h = h_{c0} + h_u + w$$

(2.3)

Here $h_u$ is defined in 2D, according to the parabolic approximation, as

$$h_u = \frac{x^2}{2R} + \frac{y^2}{2R}$$

and in 1D as

$$h_u = \frac{x^2}{2R}$$

where $R$ is the radius of sphere or cylinder.

2.2.3 Elastic Deflection

The deformation in an elastohydrodynamic contact is usually calculated using the Hertzian theory of elastic contact which was formulated by Hertz in 1881 [3]. He considered the stresses and deformations in two perfectly smooth, ellipsoidal, con-
tacting solids. He made the following assumptions: [16]

1. The materials are homogeneous and the yield stress is not exceeded.

2. No tangential forces are induced between the solids.

3. Contact is limited to a small portion of the surface such that the dimensions of the contact region are small in comparison with the radii of the ellipsoids.

4. The solids are at rest and in equilibrium.

5. The solids are semi-infinite.

If, for two contacting bodies, \( E_1, E_2 \) denotes the Young’s moduli and \( \sigma_1, \sigma_2 \) denote the Poisson’s ratios, then the reduced elastic modulus \( E_r \) is defined as

\[
\frac{1}{E_r} = \frac{1}{\pi} \left( \frac{1 - \sigma_1^2}{E_1} + \frac{1 - \sigma_2^2}{E_2} \right). \tag{2.4}
\]

With the reduced elastic modulus, one surface is assumed to be rigid and the other accounts for the total deformation of the system. The considered elastic deformation of the equivalent body surface from its undistorted shape is described by \( w \). From the Hertzian theory \( w \) is given by [17]

\[
w(x, y) = \frac{1}{E_r} \int_A \frac{p(x_1, y_1)dx_1dy_1}{\sqrt{(x-x_1)^2 + (y-y_1)^2}} \tag{2.5}
\]

where \( p \) is the fluid pressure at \((x_1, y_1)\) and acting on an elementary area \( dx_1dy_1 \) of the body surface. A complete discussion of the Hertzian contact theory can be found in Contact Mechanics by Johnson. [3]

In a one-dimensional contact, discretisation of equation \( 2.5 \) gives the deflection at point \( i \) caused by the pressure on a surface element \( j \) of half-width \( c_j \) and distance \( d_{ij} \) \[2\]

\[
w_{i,j} = \frac{p_j}{E_r} \left\{ (4c_j \ln(2b)) + (d_{ij} - c_j)\ln(d_{ij} - c_j)^2 - (d_{ij} + c_j)\ln(d_{ij} + c_j)^2 \right\} \tag{2.6}
\]

The pressure \( p_j \) is assumed to be constant over the element and \( b \) is the half length of the contacting roller. The total deflection \( w_i \) at a point \( i \) is

\[
w_i = \sum_{j} w_{i,j} \tag{2.7}
\]
The geometry of the deformed surface is expressed in relation of the deflection $w_{ref}$ of a reference point. The film thickness equation is then given by

$$h = h_c + h_u + (w - w_{ref}). \quad (2.8)$$

### 2.2.4 Cavitation

In the exit of an EHL contact the pressure drops below vapour pressure in the diverging part and the fluid cavitates and forms oil-streamers and air fingers [2]. A common model for this phenomenon is called the *Swift-Steiber boundary condition* or the *Reynolds exit condition* which sets

$$p = 0 \quad (2.9)$$

at the location $x_e$ where cavitation starts.

### 2.2.5 Energy Equation

Thermal effects can be very important in an EHL contact. Dowson [14] was the first to solve the energy equation for the fluid. For the derivation of the temperature equation it is usually assumed: [2]

- Velocity and temperature gradients are only significant across the lubricant film and are negligible along the flow direction.
- Velocity across the film is negligible.
- Thermal conductivity is constant across film.

The final form of the energy equation reads [2]

$$\nu u \theta \frac{\partial p}{\partial x} + \eta \left( \frac{\partial u}{\partial z} \right)^2 = \rho u C_p \frac{\partial \theta}{\partial x} - k \frac{\partial^2 \theta}{\partial z^2} \quad (2.10)$$

with the coefficient of thermal expansion $\nu$, lubricant specific heat at constant pressure $C_p$, thermal conductivity $k$ and temperature $\theta$ in degrees Kelvin. The terms from left to right constitute compressive heating, viscous heating, convection and conduction.

*Cheng and Sternlicht* [18] made the viscosity temperature, pressure and shear-rate dependent and employed physical boundary condition for the solids. *Yang*
allowed for the viscosity to vary across the film and derived an equivalent viscosity to be used in the Reynolds equation. He applied this method to thermal cases and found that the influence of temperature is more important than the non-Newtonian influence. Kim employed the Carslaw-Jaeger temperature boundary conditions in an thermal EHL contact and found fair agreement to experiments. Kaneta investigated the effects of thermal conductivity of the contacting surfaces which he found to be very important with a large impact on the film-shape. Kaneta also investigated the effect of compressive heating on traction force and film thickness and found that the film thickness is hardly influenced but that compressive heating as well as non-Newtonian fluid behaviour plays a very important role in the traction force. Yang, Cui, Jin and Dowson published a three part paper analysing transient elliptical contacts. In the first part with a isothermal Newtonian solver they imposed a sudden load increase which had the effect of entraining some fluid which needed to pass through the contact, resulting temporarily in a thicker film. In the second part they introduced thermal effects resulting in lower film-thicknesses due to lower viscosity and a significantly different transient behaviour of the friction coefficient. In the third part they introduced thermal effects together with non-Newtonian behaviour and compared their results with previous results. They found that non-Newtonian effects under isothermal conditions were very important compared to purely isothermal results, but were less important in a thermal analysis. The thermal effect were more dominant than the non-Newtonian effects under the conditions studied.

Solid Boundary Conditions

In an EHL contact the heat generated can be very significant and heat conduction into the bounding solids is the main mechanism of removing heat from the fluid which causes the solids to be heated up. Cheng and Sternlicht solved the energy equation for the fluid in an EHL contact and made the viscosity temperature, pressure and shear-rate dependent. They developed thermal boundary conditions for moving solids based on the work by Carslaw and Jaeger, assuming a semi-infinite body. For a one-dimensional line contact the temperature of the solids is

\[ T_{\text{cars}}(x) = \sqrt{\frac{1}{\pi \rho_s C_s k_s u_s}} \int_{-\infty}^{x} q_f(\hat{x}) \frac{d\hat{x}}{\sqrt{x - \hat{x}}} \]  

with the solid density \( \rho_s \), heat capacity \( C_s \), thermal conductivity \( k_s \), surface velocity \( u_s \) and heat flux from fluid to solid \( q_f \). Equation can be discretised for the use
on a computational mesh for the temperature of a surface element extending from \(x_{i,\text{min}}\) to \(x_{i,\text{max}}\) as

\[
T_{\text{cars},i}(x) = \sqrt{\frac{1}{\pi \rho_s C_s k_s u_s}} \sum_{i \leq x} -2 q_f(x_i) \left[ \sqrt{x - x_{i,\text{max}}} - \sqrt{x - x_{i,\text{min}}} \right] \tag{2.12}
\]

The applicability of equation 2.11 depends on the Peclet number, which is a measure of the ratio of advection to diffusion and is defined for thermal diffusion as

\[
P_e = \frac{L u}{\alpha_T} \tag{2.13}
\]

where \(L\) is the characteristic length scale, \(u\) is the velocity and \(\alpha_T\) is the thermal diffusivity of the solid defined as

\[
\alpha_T = \frac{k_s}{\rho_s C_s} \tag{2.14}
\]

The characteristic scales for an EHL contact are the contact radius \(a\), the velocity of the moving surface \(u_s\) and the thermal diffusivity of the solid \(\alpha_{T,s}\). According to Johnson [3], at large Peclet numbers \(P_e > 5\), the heat will diffuse only a short distance into the solid in the time taken for the surface to move through the heated zone. The heat flow will then be approximately perpendicular to the surface. For lower Peclet numbers than \(P_e = 5\) equation 2.11 should not be used.

For a stationary surface the Carslaw and Jaeger boundary condition is not appropriate and the predicted surface temperature would go to infinity. For a semi-infinite half-space the steady-state, discretised temperature distribution, is according to Johnson [3],

\[
T_i = T_0 + \sum_{\text{all } j} q_{s,j} S_{i,j} \tag{2.15}
\]

Here, \(T_0\) as the reference temperature, \(q_{s,j}\) as the heat-flux from fluid to solid at the surface element \(j\) and \(S_{i,j}\) as the influence coefficient of element \(j\) on the position of \(i\). \(S_{i,j}\) for a surface element \(j\) with the area \(A\) in distance \(x\) and \(y\) from position \(i\)

\[
S_{i,j} = \frac{1}{2\pi R_s} \int_A \frac{dx_1dy_1}{\sqrt{(x - x_1)^2 + (y - y_1)^2}} \tag{2.16}
\]

The Hertzian contact theory introduced in section 2.2.3 is evaluated using an integral of the same form, which means that the same discretisation can be applied. For a one-dimensional line contact equation 2.16 can be discretised for an rectangular
element with half-width $c_j$ and distance $d_j$ as

$$S_{i,j} = \frac{1}{2\pi k_s} \left\{ (4c_j \ln(2b)) + (d_j - c_j)\ln(d_j - c_j)^2 - (d_j + c_j)\ln(d_j + c_j)^2 \right\} \quad (2.17)$$

Equation 2.15 is modified to calculate the temperature in relation to a reference point with fixed temperature

$$T_i = T_0 + \sum_{all_j} q_{s,j} \left( S_{i,j} - S_{i_{ref},j} \right) \quad (2.18)$$

### 2.2.6 Fluid properties

#### Viscosity

The viscosity of lubricants depends strongly on pressure and temperature. A simple model for pressure dependency developed by Barus [29] is:

$$\eta_{barus} = \eta_0 e^{\alpha_p p} \quad (2.19)$$

where $\eta_0$ is the atmospheric viscosity and $\alpha_p$ is the pressure viscosity coefficient. For mineral oils $\alpha_p$ is generally in the range between $10^{-8}$ and $2 \cdot 10^{-8} Pa^{-1}$. [30].

A simple model for the temperature dependency after Reynolds [9] is:

$$\eta_{reynolds} = \eta_0 e^{-\beta (T - T_0)} \quad (2.20)$$

with $T$ as the fluid temperature and $T_0$ at the reference temperature for $\eta_0$. $\beta$ is a thermoviscous constant. For example for Shell Turbo33 oil is $\beta = 0.0476/K$. [2]

A more realistic model including both pressure and temperature dependence by Roelands [12] and developed by Houpert [31] is: [2]

$$\eta_{houpert} = \eta_{roelands} \exp(-\beta^* (T - T_0)) \quad (2.21)$$

with

$$\beta^* = (\ln(\eta_0) + 9.67) \left( 1 + 5.1 \cdot 10^{-9} p \right)^2 \frac{S_0}{T_0 - 138K} \quad (2.22)$$

and

$$\eta_{roelands} = \eta_0 \left( \alpha_p^* p \right) \quad (2.23)$$
Here

\[ \alpha_p^*p = \left\{ (\ln(\eta_0) + 9.67) \left[ \left( 1 + \frac{p}{p_0} \right)^Z - 1 \right] \left( \frac{T - 138K}{T_0 - 138K} \right)^{-S_0} \right\} \]  

(2.24)

\( Z \) and \( S_0 \) are given by

\[ Z = \frac{\alpha_p}{5.1 \cdot 10^{-9} \ln(\eta_0) + 9.67} \]  

(2.25)

and

\[ S_0 = \frac{\beta(T_0 - 138)}{\ln(\eta_0) + 9.67} \]  

(2.26)

Density

A widely used density-pressure relation for lubricating oil was developed by Dowson:

\[ \rho(p) = \rho_0 \frac{5.9 \cdot 10^8 + 1.34p}{5.9 \cdot 10^8 + p} \]  

(2.27)

where \( \rho_0 \) is the atmospheric density and \( p \) is in \( Pa \).

Non-Newtonian Behaviour

In elastohydrodynamic lubrication, fluids do not follow Newtonian behaviour and the Newtonian model overpredicts the shear-stress found in experiments as shown by Johnson and Tevaarwerk. They proposed the Ree-Eyring model instead, which includes a non-linear stress-strain relationship. If the fluid is viscoelastic, the total shear strain rate is:

\[ \dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}_v \]  

(2.28)

If the fluid is Newtonian with viscosity \( \eta \) and viscous shear strain \( \dot{\gamma}_v \), then the shear stress is:

\[ \tau = \eta \dot{\gamma}_v \]  

(2.29)

Considering the fluid as linear elastic and with \( G \) as the elastic shear modulus, the elastic shear strain rate is:

\[ \dot{\gamma}_e = \frac{\tau}{G} \]  

(2.30)
Shear-thinning occurs according to

\[ \dot{\gamma}_v = \frac{\tau_0}{\eta} \sinh \left( \frac{\tau}{\tau_0} \right), \]  

(2.31)

The overall shear rate is therefore

\[ \dot{\gamma} = \frac{\dot{\tau}}{G} + \frac{\tau_0}{\eta} \sinh \left( \frac{\tau}{\tau_0} \right). \]  

(2.32)

The reference stress \( \tau_0 \) is called *Eyring stress*, which marks the boundary between Newtonian and non-Newtonian behaviour. Conry [10] developed a Reynolds equation incorporating the findings of Johnson and Tevaarwerk by replacing the fluid viscosity with an effective viscosity, which is calculated by averaging the effect of shear-thinning across the film.

For a CFD approach the shear-rate dependency can be expressed locally. In equation 2.32 the elastic component is dropped, as *Bair* [33] has shown that a time-dependency in the viscous response for an ehl-contact is unlikely. This leads to

\[ \dot{\gamma} = \frac{\tau_0}{\eta_0} \sinh \left( \frac{\tau}{\tau_0} \right). \]  

(2.33)

\( \tau \) is the shear-stress of the fluid and can be expressed as

\[ \tau = \eta \dot{\gamma} \]  

(2.34)

Equation 2.34 inserted into equation 2.33 gives shear-thinning viscosity of the fluid

\[ \eta_{\text{Eyring}} = \frac{\tau_0}{\dot{\gamma}} \sinh^{-1} \left( \frac{\eta_0 \dot{\gamma}}{\tau_0} \right). \]  

(2.35)

For the viscosity model employed in the following calculations \( \eta_0 \) is replaced by \( \eta_{\text{Houpert}} \) of equation 2.21 in order to avoid division by zero. The shear-thinning part is only used if the shear-rate is larger than \( \dot{\gamma}_{\text{min}} = 10^{-8}/s \) for numerical reasons, leading to the final formulation of viscosity.

\[ \eta(T, p, \dot{\gamma}) = \begin{cases} \eta_{\text{Houpert}} & \text{for } \dot{\gamma} < \dot{\gamma}_{\text{min}} \\ \frac{\eta_{\text{Houpert}}}{\dot{\gamma}} \sinh^{-1} \left( \frac{\eta_{\text{Houpert}} \dot{\gamma}}{\tau_0} \right) & \text{for } \dot{\gamma} \geq \dot{\gamma}_{\text{min}} \end{cases} \]  

(2.36)
2.3 Numerical Methods

The equations in section 2.2 which describe the elastohydrodynamic problem can be solved numerically. Elcoate provided a clear overview of relevant methods entitled in *On the coupling of the elastohydrodynamic problem* [34]. There are many different flavours of discretisation and solution strategies available. The currently used methods may be categorised as

- Inverse
- Forward iterative
- Fully coupled
- Partly coupled.

2.3.1 Inverse Method

Downson and Higginson [13] gave the first full numerical solutions to the 1D cylindrical EHL problem. To start the calculation a pressure distribution is assumed in the contact, which might be the pressure of a dry Hertzian contact. The computational domain is split into two regions. The first region, with relatively low pressure, is calculated using a forward iterative method, of iteratively calculating the Hertzian deflection and solving the Reynolds equation (2.1). The second region, where pressure is high, is calculated employing the inverse method. For this, the Reynolds equation (2.1) is inverted to calculate the film thickness given a pressure distribution. This film thickness is compared with the thickness provided by the Hertzian deflection and the pressure distribution is corrected until the two shapes agree.

This procedure is not an easy one and the time taken depends on the experience of the person performing it. [35]. Therefore it has not received general application in spite of its stability and ability to solve demanding problems [34].

2.3.2 Forward iterative methods

The forward iterative method is the simplest approach. It is described in detail by Gohar [2]. In this method, the fluid pressure calculated by equation (2.1) usually discretised using the finite difference method on a uniform mesh, the Hertzian deflection according to equation (2.5), the film-thickness from equation (2.3) and the
fluid properties are calculated one after another. The cavitation treatment is usually incorporated into the solver of the fluid pressure equation, (e.g. Gauss-Seidel). This method is easy to implement but it becomes unstable at high pressures. To enhance stability, the pressure and/or deflection calculation can be underrelaxed.

**Multilevel**

The multilevel, or multigrid method is a more sophisticated forward iterative method. Simple relaxation alone does not provide enough stability to achieve the very high pressure reached in many engineering applications. This is because it has good error smoothing properties on error components with wavelengths comparable to the grid size, but errors with wavelengths large compared to the mesh size converge only slowly. In order to speed up the calculation, the solution of a fine target mesh is mapped onto a coarse grid, which enables smoothing of errors with bigger wavelengths. There can be several different grids, in order to smooth the errors of the corresponding wavelengths efficiently. The simplest order of calculation on these grids is the $V$-cycle, where the first solution is obtained on the finest mesh. Subsequently, the solution is interpolated onto a coarser mesh and the problem is solved again. From the coarsest mesh the process is reversed until the fine mesh is reached. This is necessary to eliminate errors due to interpolation. Venner has described this approach in detail in *Multilevel methods in lubrication* [30]. A general discussion of the multigrid approach can be found in *Multigrid* by Trottenberg. [36]

**Multilevel Multi-integration**

The multilevel method speeds up the calculation and helps to improve stability. To obtain further reductions in computational time, this method can be used in conjunction with the multi-integration approach for the calculation of the deformation integral (2.5). The discretised form of this is

$$w_i = \sum_{\text{all } k} g_{k,i}p_k$$

(2.37)

with $w_i$ as deflection, $g_{k,i}$ as the influence coefficient and $p_k$ the pressure. To evaluate this sum $O(n^2)$ operations are needed, where $n$ is the number of grid points. This is computationally very expensive and can be reduced by performing integration on coarser grids. To ensure accuracy, the influence coefficients have to
be smooth, so they can be approximated accurate enough on a coarser grid. The general procedure is: 

- Interpolate the influence coefficients, so that the integral can be expressed correctly on the coarse grid.
- Do the coarse grid summation.
- Interpolate the results back to the fine grid.

This process can involve several levels of grid density with subsequent interpolation to the coarsest grid and back-interpolation to the finest level. If the influence coefficient function $g$ is smooth, only one step from the fine to the coarse grid is needed. For a non-smooth function of coefficients a more elaborate approach is needed. The multi-integration method can be used to reduce the number of operations from $O(n^2)$ to $O(n)$. For more details refer to Venner [30] or Gohar [2].

### 2.3.3 Fully coupled methods

The deflection equation 2.3 and the Reynolds equation 2.1 have the film thickness $h$ as a variable in common. This allows these equations to be discretised for example with a finite difference method and couple together, resulting in a full matrix problem. This approach is capable of obtaining solutions to demanding thin-film rough surface problems. However, these capabilities are at the expense of computational time and the equations are not easy to solve [34].

Hughes, Elcoate and Evans [37] developed a fully coupled finite difference approach which can deliver stable solutions to a maximum pressure of 4.0 GPa for a one-dimensional line contact.

### 2.3.4 Partly coupled methods

Following the development of the aforementioned method fully coupled method [37], Hughes et al. [34] explored the possibility of partial coupling. In a conventional forward iterative method, equation 2.3 is evaluated using the pressure from the previous iteration cycle in equation 2.5. To implement partial coupling within the iterative scheme the film-thickness at grid point $i$ is expressed as

$$ h_i = h_{c0,i} + h_{u,i} + w_{n,i} + w_{f,i} - w_0 $$

(2.38)
where \( w_n \) is the deflection caused by grid-points near grid-point \( i \) using the current pressures, \( w_f \) is the deflection caused by all other grid-points using the previous pressures and \( w_0 \) is the deflection of a reference point. Unfortunately this produces oscillations with a wavelength equal to the bandwidth of the near deflection term.

**Differential deflection**

In order to overcome the difficulties of the partly coupled method above, Evans and Hughes [38] studied the nature of the Hertzian deflection calculation (equation 2.5). Equation (2.5) can discretised on a uniform and expressed as a simple summation, where \( i \) is the index of a node and \( g_i \) is the influence coefficient, which is only dependent on grid geometry:

\[
 w_i = \sum_{all\ k} g_{k-i} p_k
\]

(2.39)

They discovered, when evaluating second derivative of deflection \( \nabla^2 w \), that the influence coefficients \( f_i \) for this differential deflection term decay rapidly with the distance from grid-point \( i \).

\[
 \nabla^2 w_i = \sum_{all\ k} f_{k-i} p_k
\]

(2.40)

If the decay of coefficients \( f_i \) and \( g_i \) are compared, then \( f_i \) is at least an order of magnitude smaller by the second mesh point and two orders of magnitude smaller by the eighth mesh point. This method is available for line and point contact and Evans demonstrated it has good accuracy compared with analytic solutions.

The application of the differential deflection method to a line contact problem and the numerical implementation are outlined in a paper by Hughes, Elcoate and Evans [39]. They have shown the method to be accurate, efficient and stable to pressures up to 4.0 GPa. The time savings in comparison with previous fully coupled methods can be more than three orders of magnitude.

**2.4 Closure**

In this chapter the traditional Reynolds equation-based approach for modelling elastohydrodynamic lubrication was introduced. This includes the Reynolds equation for the fluid film, the commonly used fluid property relations, the elastic Hertzian contact theory and the energy equation together with appropriate ther-
mal boundary conditions.
Chapter 3

Fluid Modelling in CFD

3.1 Introduction

In this chapter the governing equations of the CFD method are introduced and
the finite volume method (FVM) is described. A computational mesh convergence
study for a two-dimensional line contact is performed. Cavitation is a very impor-
tant phenomenon occurring in EHL contact. In the Reynolds approach cavitation
is usually simply modelled by modifying the Gauss-Seidel matrix solver for the
Reynolds equation such that the pressure is forced to be greater or equal than
zero. In a CFD approach this is not possible as the pressure equation is used to
maintain continuity and any direct tampering with the matrix solver would lead to
continuity errors. Furthermore, it is desirable to move to a better understanding of
the cavitation zone in EHL contacts. Cavitation modelling is a huge research field
in its own right. In this chapter a short literature research on cavitation is given
and two cavitation models are discussed and adapted to meet the demands of an
EHL contact calculation.

3.2 Governing Equations

Fluid flow is mathematically described by the conservation of mass, momentum
and energy. The general form of a conservation equation for a flow quantity $\phi$ is

$$\frac{\partial \rho \phi}{\partial t} + \nabla \cdot (\rho \mathbf{u} \phi) - \nabla \cdot (\rho \Gamma_\phi \nabla \phi) = S_\phi \tag{3.1}$$

where $\rho$ is density, $t$ is time, $\mathbf{u}$ is velocity, $\Gamma_\phi$ is diffusivity and $S_\phi$ is a source term.

The transport equation for the conservation of mass, or continuity equation, is
derived by setting $\phi = 1$ in equation [3.1] and not having mass source terms. This leads to

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0$$

(3.2)

The momentum equation, neglecting gravitational effects, according to Bird [41] is

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) - \nabla \cdot \mathbf{\tau} = -\nabla p$$

(3.3)

where $\mathbf{\tau}$ is viscous stress tensor given by

$$\mathbf{\tau} = -\mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \frac{2}{3} \mu I \nabla \cdot \mathbf{u}$$

(3.4)

where $\mu$ is the viscosity of a Newtonian fluid. Lubricants inside an EHL contact do not behave in a Newtonian way and the fluid viscosity will be adapted for non-Newtonian behaviour in chapter [5].

The energy equation may be expressed in terms of enthalpy $H$ and is given by

$$\frac{D(\rho H)}{Dt} = \nabla \cdot k \nabla T - \nabla \cdot \mathbf{\tau} + \frac{Dp}{Dt}$$

(3.5)

where $k$ is the thermal conductivity.

### 3.3 Finite Volume Method

For all calculations the OpenFOAM package [42] is used which employs the finite volume method. The FVM subdivides the flow domain into a finite number of smaller non-overlapping control volume. The transport equations are then integrated over each these control volumes by approximating the variation of flow properties between mesh points with differencing schemes. This section gives only a brief overview on the FVM [43] [44], for a complete discussion on the FVM the reader is referred to *Computational Methods for Fluid Dynamics* by Ferziger and Perić [40].

#### 3.3.1 Discretisation of the Solution Domain

The desired solution domain is broken up, *discretised*, into a number of cells, or control volumes. These are contiguous, meaning that they do not overlap one another and completely fill the domain. Generally variables are stored at the cell centroid, although they may be stored on faces or vertices. A cell is bounded by a
set of flat faces with no limitation on the number of faces or their alignment, which can be called "arbitrarily unstructured". Two neighbouring cells must only share one face, which is then called an "internal face". A face belonging only to one cell is called a "boundary face". The minimum information required to define a mesh consists of

**Points**, which are defined by their position in three-dimensional space.

**Faces**, which are defined by a list of points.

**Cells**, which are defined by a list of faces.

**Boundary patches**, which are defined by a list of boundary faces, with each face being a member of only one boundary patch. The boundary patches have to contain all boundary faces.

### 3.3.2 Discrete equations

The partial differential equation 3.1 is transformed into an algebraic expression, which can be expressed as

\[
M x = b
\]  

(3.6)

where \( M \) is a square matrix, \( x \) the vector of the dependent variable and \( b \) is the source vector. *Finite Volume* (FV) discretisation of each term is formulated by first integrating the term over a cell volume \( V \). Most spatial derivatives are converted to integrals over the cell surface \( S \) bounding the volume using Gauss’s theorem

\[
\int_V \nabla \phi = \int_S dS \phi
\]  

(3.7)

where \( S \) is the surface area vector, \( \phi \) can represent any variable. Implicit terms constitute the matrix \( M \), explicit terms constitute the source vector \( b \).

**The Diffusion Term**

The diffusion term is integrated over a control volume and linearised as follows:

\[
\int_V \nabla \cdot (\Gamma \nabla \phi) dV = \int_S d(S \cdot (\Gamma \nabla \phi)) = \sum_f \Gamma_f S_f \cdot (\nabla \phi)_f
\]  

(3.8)
That can be discretized when the length vector \( \mathbf{d} \) between the centre of the cell of interest \( P \) and the centre of a neighbouring cell \( N \) is orthogonal to the face:

\[
\mathbf{S}_f \cdot (\nabla \phi)_f = |\mathbf{S}_f| \frac{\phi_N - \phi_P}{|\mathbf{d}|} \tag{3.9}
\]

For non-orthogonal meshes an additional explicit term is introduced. The reader is referred to the Programmers Guide [43] for more information.

**The Convection Term**

The convection term is integrated over a control volume and linearised as follows:

\[
\int_V \nabla \cdot (\rho \mathbf{U} \phi) dV = \int_S d(\mathbf{S} \cdot (\rho \mathbf{U} \phi)) = \sum_f \mathbf{S}_f \cdot (\rho \mathbf{U})_f \phi_f = \sum_f F_f \phi_f \tag{3.10}
\]

The face field \( \phi_f \) can be evaluated using a variety of schemes.

**Central differencing** is second-order accurate but unbounded, meaning that the error of discretisation reduces with the square of the grid spacing and that the limits of \( \phi \) are not necessarily preserved.

\[
\phi_f = f_x \phi_p + (1 - f_x) \phi_N \tag{3.11}
\]

where \( f_x \equiv f_N/P_N \).

**Upwind differencing** is first-order accurate and bounded and determines \( \phi_f \) from the direction of flow.

\[
\phi_f = \begin{cases} 
\phi_P & \text{for } F \geq 0 \\
\phi_N & \text{for } F < 0
\end{cases} \tag{3.12}
\]

Those two schemes can be blended in order to preserve boundedness with reasonable accuracy and there are many more schemes implemented which might be investigated.
The Gradient Term

The gradient term described here is an explicit term. Usually it is evaluated using the Gauss integration by applying the Gauss theorem to the volume integral:

$$\int_V \nabla \phi dV = \int_S d(S\phi = \sum_f S_f \phi_f)$$  \hspace{1cm} (3.13)

There are more ways to evaluate the gradient term, refer to the Programmer’s Guide [43].

The Time Derivative

The time derivative $\partial/\partial t$ is integrated over a control volume as follows:

$$\frac{\partial}{\partial t} \int_V \rho \phi dV$$  \hspace{1cm} (3.14)

That is discretised by using:

**new values** $\phi^n \equiv \phi(t = \Delta t)$ at the time step being solved for

**old values** $\phi^o \equiv \phi(t)$ that were stored from the previous time step

Euler implicit is the only scheme used in this study, which is first order accurate in time, meaning that the discretisation error reduces with smaller time-steps. It is discretised as follows:

$$\frac{\partial}{\partial t} \int_V \rho \phi dV = \frac{(\rho_p \phi_p V)^n - (\rho_p \phi_p V)^o}{\Delta t}$$  \hspace{1cm} (3.15)

### 3.3.3 Temporal Discretisation

The treatment of time derivatives is explained in the section above. But the spatial derivatives in a transient problem also need some consideration as $\phi$ is function of space and time and spatial and the spatial derivatives are averaged over one or more timesteps. If all spatial terms are denoted as $A \phi$, where $A$ is any spatial operator, e.g. Laplacian, then a transient partial differential equation (PDE) can be expressed as

$$\int_t^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_V \rho \phi dV + \int_V A \phi dV \right] dt = 0$$  \hspace{1cm} (3.16)
Using the Euler implicit method of equation 3.15, the first term can be expressed as

\[
\int_t^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_V \rho \phi dV \right] dt = \frac{\left( \rho_P \phi_P V \right)^n - \left( \rho_P \phi_P V \right)^o}{\Delta t} 
\] (3.17)

The second term of equation 3.16 can be expressed as

\[
\int_t^{t+\Delta t} \left[ \int_V A \phi dV \right] dt = \int_t^{t+\Delta t} A^* \phi dt 
\] (3.18)

where \( A^* \) represents the spatial discretisation of \( A \). That integral can be discretised as:

**Euler implicit** taking only current values \( \phi^n \), is first order accurate in time, guarantees boundedness and is unconditionally stable.

\[
\int_t^{t+\Delta t} A^* \phi \ dt = A^* \phi^n \Delta t 
\] (3.19)

**Explicit** taking only values \( \phi^o \) from the previous timestep, guarantees boundedness and is first order accurate in time.

\[
\int_t^{t+\Delta t} A^* \phi \ dt = A^* \phi^o \Delta t 
\] (3.20)

It is unstable if the Courant number \( Co \) is greater than one. The Courant number is defined as

\[
Co = \frac{U_f \cdot d}{|d|^2} \Delta t 
\] (3.21)

where \( U_f \) is the velocity of the flow or velocity of a wave front for the acoustic Courant number. \( d \) is the length vector between two neighbouring cell centres.

**Crank Nicholson** is taking a mean of current values \( \phi^n \) and old values \( \phi^o \). It is second order accurate in time, unconditionally stable, but does not guarantee boundedness.

\[
\int_t^{t+\Delta t} A^* \phi \ dt = A^* \left( \frac{\phi^n + \phi^o}{2} \right) \Delta t 
\] (3.22)

Of the time schemes introduced **Euler implicit** is the most stable scheme and is is the only one used in this study.


3.3.4 Boundary Conditions

The boundaries of the computational domain define the physical problem. For each independent variable boundary conditions need to be specified. The aim is to emulate the physical world as closely as possible. Boundary conditions can be divided into 2 types:

**Dirichlet** prescribes the value of the dependent variable on the boundary and is termed 'fixed value'

- In cases where the discretisation requires the boundary value on a boundary face $\phi_f$, e.g. in the convection term in equation 3.10, the specified value is taken.
- In terms where the face gradient $(\nabla \phi)_f$ is required, e.g. Laplacian, it is calculated using the specified boundary face value $\phi_b$ and the cell centre value $\phi_P$. $S_f$ denotes the face area vector.

$$S_f \cdot (\nabla \phi)_f = |S_f| \frac{\phi_b - \phi_P}{|d|}$$ (3.23)

**Neumann** prescribes the gradient of the variable normal to the boundary and is termed 'fixed gradient'. The fixed gradient boundary condition $g_v$ is a specification on the inner product of the gradient and unit normal to the boundary, or

$$g_b = \left( \frac{S_b}{|S_b|} \cdot \nabla \phi \right)_b$$ (3.24)

with $S_b$ being the face area vector of the boundary face

- When discretisation requires the value on a boundary face $\phi_f$, the cell centre value is interpolated to the boundary by

$$\phi_f = \phi_P + d_n \cdot (\nabla \phi)_f = \phi_P + |d_n| g_b$$ (3.25)

with $d_n$ as the vector between boundary face and cell centre and being normal to the boundary face.

- When the discretisation requires the face gradient to be evaluated, the specified gradient can be taken directly

$$S_f \cdot (\nabla \phi)_f = |S_f| g_b$$ (3.26)
Physical Boundary Conditions

Boundary conditions need to be specified which reflect the physical behaviour of the fluid. Since there are many possible boundary conditions, only the applied ones are introduced.

No-slip impermeable wall  The velocity of the fluid is equal to that of the wall. The pressure and density boundary conditions are zero gradient, since the flux through the wall is zero.

Symmetry plane  In a symmetry plane the component of the gradient normal to the plane is zero. The components parallel to it are projected to the boundary face from the inside of the domain.

Total pressure  The total pressure \( p_0 = p + \frac{1}{2} \rho |U|^2 \) is fixed, when \( U \) changes, \( p \) is adjusted accordingly.

3.4 Grid Study

Before introducing cavitation models, a grid study of a two-dimensional half-cylinder on flat plane is performed with a non-cavitating, incompressible solver. The reason is that, for the Reynolds equation outlined in chapter 2, an analytical solutions exists, which can be used to determine the quality of the solution. The pressure gradients encountered in a non-cavitating solution are similar to the pressure gradients of a cavitating problem, so that this study can serve as a good indication of the mesh-quality for a cavitating case.

The domain consists of a half-cylinder on flat surface in two dimensions as shown in figure 3.1. The parameters are summarised in table 3.1. The viscosity is that of a typical lubricant in a bearing. The viscosity is not yet piezo-viscous at the resulting pressure level.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder radius</td>
<td>( R = 10mm )</td>
</tr>
<tr>
<td>length of domain</td>
<td>( D = 120mm )</td>
</tr>
<tr>
<td>surface velocity</td>
<td>( v = 1m/s )</td>
</tr>
<tr>
<td>film thickness</td>
<td>( h_0 = 10^{-7}m )</td>
</tr>
<tr>
<td>kinematic viscosity</td>
<td>( \nu = 10^{-5}m^2/s )</td>
</tr>
</tbody>
</table>

Table 3.1: Case parameter
3.4.1 Numerical Setup

For the calculation, the *simpleFoam* solver of the OpenFOAM packages was used, which is a steady-state solver for incompressible, isoviscous fluids. The flow is assumed to be laminar. The governing equations are the continuity equation and a simplified momentum equation:

\[
\nabla \cdot (\mathbf{uu}) + \nabla \cdot \nu \nabla \mathbf{u} = -\nabla (p) \quad (3.27)
\]

As the discretisation scheme, a central differencing scheme was employed. The underrelaxation factors are 0.3 for the pressure equation and 0.7 for the momentum equation. The solver used for the pressure equation is *Incomplete-Cholesky preconditioned conjugate gradient* (ICCG) and for the momentum equation the solver used is *Incomplete-Cholesky preconditioned biconjugate gradient* (BICCG). The number of iterations was set to 400 for the base case and for the other cases the number of iterations is linear dependent on scaling factor in the x direction.

3.4.2 Base Mesh

For the start of the grid study a rather coarse grid was chosen with 2230 hexahedra in total. In the z direction there are 10 cells between cylinder and moving wall, and in the x direction 208 cells are used. The cells in the z direction in the gap are equally spaced. In the x direction the cells length is expanding from the middle to the outer boundaries with a constant cell-to-cell expansion ratio of \( r = 1.1 \). The maximum aspect ratio \((\Delta x/\Delta y)_{\text{cell}}\) is 500. The base mesh is shown in figure 3.2 and, at a larger scale, in figure 3.3. The other grids are refinements of this base mesh.
mesh with scaling factors 2, 4 and 8 in the x and the z direction.

Figure 3.2: Grid study - base mesh (1)

Figure 3.3: Grid study - base mesh (2)

3.4.3 Results

The numerical results are compared to the analytical solution for the pressure, which is called Full Sommerfeld, as given in: [2]

\[
p(x) = \frac{2 U \eta x}{h^2 \left( \frac{1+x^2}{2Rh} \right)^2}
\]  

(3.28)

Table 3.2 shows the standard deviation divided by one thousand of the numerical solution to the full Sommerfeld solution normalised by the pressure range \( p_{\text{range}} = p_{\text{max}} - p_{\text{min}} = 58094.75 \text{m}^2\text{s}^{-2} \) of all cases. In the row ZScale = 1 the standard deviation is rising although the resolution x-direction is increased. This points to
the importance of an appropriate aspect ratio for the computational cells. Increasing the resolution in one direction does not necessarily improve the accuracy of the solution. If the cases with 1/1, 2/2 and 4/4 scaling are considered, the error is reduced to a quarter for each step. This is consistent with having a second-order differencing scheme, where the error should be reduced with the grid-spacing squared. Beyond the 4/4 scaling case the error does not change much more, as the point of grid-independent solution has been reached. Therefore, it would not make sense to use grids with a much higher resolution than in the 4/4 case.

<table>
<thead>
<tr>
<th>XScale=1</th>
<th>XScale=2</th>
<th>XScale=4</th>
<th>XScale=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZScale=1</td>
<td>1.776</td>
<td>2.278</td>
<td>2.466</td>
</tr>
<tr>
<td>ZScale=2</td>
<td>1.211</td>
<td>0.442</td>
<td>0.573</td>
</tr>
<tr>
<td>ZScale=4</td>
<td>1.504</td>
<td>0.307</td>
<td>0.108</td>
</tr>
<tr>
<td>ZScale=8</td>
<td>1.790</td>
<td>0.361</td>
<td>0.106</td>
</tr>
</tbody>
</table>

Table 3.2: Standard deviation [%]

In figure 3.5 the pressure distribution of the base case compared to the analytical solution is shown. The solution is point symmetric around the origin and the pressure is zero at the origin.

In figure 3.4 standard deviation of the grids in the diagonal of table 3.2 are compared. The maximum deviation to the analytical solution occurs at positions with high pressure gradient near location of maximum or minimum pressure. The shapes of the curves in case of 1/1, 2/2 and 4/4 scaling are similar, with the magnitude of the deviation dropping.

3.5 Literature Research on Cavitation

In this section a brief summary of the literature about cavitation and its modelling in bearings is given. The following textbooks are often quoted by other authors who are working on the problem of cavitation. The oldest is *Cavitation* by Knapp, Daily and Hammitt in 1970 [45] which was the first book summarising the effect of cavitation systematically. It covers a wide range of cavitation phenomena including experiments, bubble growth and collapse, describes the influence on the flow field and the process of cavitation damage. Hammitt followed with *Cavitation and Multiphase Flow Phenomena* in 1980 [46] which summarised the former book and updated it. Another important book is *One-dimensional Two-Phase Flow* [47] by Graham B. Wallis published in 1969 which deals with two-phase phenomena.
He derived general equations for two-phase flow, developed models for mixing fluid properties and relate these to experimental evidence. The latest book dealing solely with cavitation phenomena is *Cavitation* [48] by Young, published in 1999, which gives an introduction in the field of cavitation covering recent work in modelling cavitation and experiments. He defines cavitation as the formation and activity of bubbles (or cavities) in a liquid and distinguishes four different kinds of cavitation:

1. **Hydrodynamic cavitation** is produced by pressure variations in a flowing liquid due to the geometry of the system.
2. **Acoustic cavitation** is produced by sound waves in a liquid due to pressure variations.
3. **Optic cavitation** is produced by photons of high intensity (laser) light rupturing in a liquid.
4. **Particle cavitation** is produced by any other type of elementary particles, e.g. a proton, rupturing as in a bubble chamber.
In this study we are only concerned with hydrodynamic cavitation as it occurs in bearings. Young divides hydrodynamic cavitation into three sub-classes:

1. *Travelling* cavitation occurs when cavities or bubbles form in the liquid, and travel with the liquid as they expand and subsequently collapse.

2. *Fixed* cavitation occurs when a cavity or pocket attached to the rigid boundary of an immersed body or a flow passage forms, and remains fixed in position in an unsteady state.

3. *Vortex* cavitation occurs in the cores of vortexes which form in regions of high shear, and often occurs on the blade tips of ship’s propellers - hence the name ’tip’ cavitation.

For ball bearings, experimental evidence [49] shows that we are dealing with fixed cavitation, where oil fingers are formed, separated by pockets of gas. Whether that gas is vapour or in dissolved gas escaping from the liquid is unclear. It should be noted that this does not always occur at above zero pressure since liquids can support some negative pressure. Brown found large negative pressure under a lubricated piston ring [50] with a peak value of $-0.78\, MPa$, which had an important effect of reducing minimum film thickness. Kaneko investigated porous journal bearings and found negative film pressures both, numerically and experimentally, of about $-10\, kPa$ [51]. Wissussek [52] found negative pressures in a radial sliding bearing of up to $-0.2\, MPa$.

Figure 3.5: Pressure of base case
3.5.1 Cavitation Modelling

Various cavitation models have been proposed for use in computational fluid dynamics. A common approach comprises the homogeneous equilibrium models. For these, a mixture density is introduced and a single set of mass and momentum equations are solved. A large variety of methods for calculating the mixture density have been suggested [53] [54] [55]. Delannoy [56] introduced one of the first homogenous models with a simple relationship between pressure and density but experienced problems with liquid/vapour density ratios higher than 1:100. Kubota et al. [57] provided one of the first homogeneous flow models, which was based on the assumption of that the fluid is a mixture of very small, spherical bubbles and where the growth and collapse of these bubbles is modelled by using a modified Rayleigh equation. Avva et al. [58] assumes homogeneous flow in local thermodynamic equilibrium and formulates a mixture energy equation. The volume fraction of vapour is calculated from the mixture enthalpy and saturated liquid and vapour enthalpies. Schmidt et al. [59] also used the mixture energy equation in local equilibrium as a starting point. He neglected thermal conduction and viscous work forces and assumed isentropic compression in the energy equation and developed a relation between pressure and density by further assuming a homogeneous flow of fine dispersed bubbles in liquid. Schmidt applied this model to a variety of nozzle geometries and successfully predicted discharge coefficients and exit velocities.

The aforementioned barotropic models have improved over time in terms of their ability to replicate experiments and have been successfully applied to nozzles and hydrofoils. Unfortunately these applications are characterised by high Reynolds number, high Weber number and negligible viscous effects. None of these assumptions can be applied to the cavitation phenomena in bearings where there are comparably low Reynolds and Weber number and as will be shown later in this chapter, viscous effects are very important and viscous heating has been observed. However, it has yet to be shown in comparison with experiments or with more sophisticated cavitation models whether the use of homogenous equilibrium models in bearings are appropriate. For example, Ye et al. [60] developed a method for the direct simulation of bubble dynamics with large density ratios and phase change and which is capable of treating the interface as a sharp discontinuity. He applied this to Reynolds numbers smaller than one hundred and Weber numbers smaller than twenty. It would be probably too expensive to use this in connection with elastic deformation, but it might be used to identify deficiencies in the
cavitation models currently employed.

### 3.6 Isentropic Cavitation Model

The cavitation model introduced in this chapter, developed by Weller [61], will be named *isentropic cavitation model* as it follows in principle the approach of Schmidt et al. [59], assuming isentropic phase change. He simplifies the mixture energy equation 3.5 assuming homogeneous and thermodynamic equilibrium of the liquid and vapour phases. Based on an order of magnitude analysis, Schmidt neglects the terms for thermal conductivity and viscous dissipation and replaces $\rho D\overline{h}$ by $a^2 Dp$ assuming entropy production to be zero, which leads to the relationship

$$a^2 D\rho = Dp$$  \hspace{1cm} (3.29)

where $a$ is the sonic velocity of the mixture. The speed of sound is calculated according to Wallis [47] via the expression below, which is strictly valid for homogeneous flows such as a fine dispersion of bubbles in a liquid. As a very rough estimate he places the transition from bubbly to slug flow at 10 percent of vapour fraction for pure liquids in one-dimensional flow.

$$\frac{1}{a^2} = (\alpha \rho_v + (1 - \alpha)\rho_l) \left( \frac{\alpha}{\rho_v a_v^2} + \frac{1 - \alpha}{\rho_l a_l^2} \right)$$  \hspace{1cm} (3.30)

Schmidt integrates equation 3.29 using 3.30 and derives an explicit function for the density of the mixture $\rho = f(p)$. In the cavitation model presented, equation 3.29 is put directly into the pressure equation, instead of using an explicit function for density.

The density of vapour is given by

$$\rho_v = \max\left(\frac{p}{a_v^2}, \rho_{v,sat}\right)$$  \hspace{1cm} (3.31)

the density of liquid is

$$\rho_l = \rho_{l,sat} + \frac{p - p_{vapour}}{a_l^2},$$  \hspace{1cm} (3.32)

the vapour fraction is defined as

$$\alpha = \frac{\rho_l - \rho}{\rho_l - \rho_v}.$$  \hspace{1cm} (3.33)
The mixture dynamic viscosity is interpolated between liquid viscosity and the vapour viscosity:

\[ \mu = \alpha \mu_v + (1 - \alpha) \mu_l \]  

(3.34)

The conservation of mass is given by equation [3.2] and the conservation of momentum is given by equation [3.3], assuming both phases travelling at the same velocity.

### 3.6.1 Solution Procedure

The discretised Laplacian equation for calculating the pressure field is derived as follows. First, expanding the continuity equation gives

\[ \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0 \]  

(3.35)

Replacement of the substantial derivative of the density with the isentropic compression produces

\[ \frac{1}{a^2} \frac{\partial p}{\partial t} + \frac{1}{a^2} \mathbf{u} \cdot \nabla p + \rho \nabla \cdot \mathbf{u} = 0 \]  

(3.36)

The discretised momentum equation can be expressed in the form

\[ \mathbf{A} \cdot \mathbf{u} = \mathbf{H}(\mathbf{u}) - \nabla p \]  

(3.37)

where \( \mathbf{A} \) contains the diagonal matrix coefficients and \( \mathbf{H}(\mathbf{u}) \) contains all off-diagonal matrix coefficients multiplied by their corresponding velocities. The velocity predictor without contribution of the pressure gradient is given by

\[ \hat{\mathbf{u}} = \mathbf{A}^{-1} \mathbf{H} \]  

(3.38)

With equation (3.37) and (3.38) inserted into (3.36):

\[ \frac{1}{\rho a^2} \frac{\partial p}{\partial t} + \frac{1}{\rho a^2} \hat{\mathbf{u}} \cdot \nabla p + \nabla \cdot \hat{\mathbf{u}} - \nabla \cdot \mathbf{A}^{-1} \nabla p = 0 \]  

(3.39)

In order to have the divergence of the velocity, which gives a strong conservative implementation, \( \frac{1}{\rho a^2} \hat{\mathbf{u}} \cdot \nabla p \) is replaced by \( \nabla \cdot \frac{1}{\rho a^2} \hat{\mathbf{u}} p - p \nabla \cdot \frac{1}{\rho a^2} \hat{\mathbf{u}} \) and leads to:

\[ \frac{1}{\rho a^2} \frac{\partial p}{\partial t} + \nabla \cdot \frac{1}{\rho a^2} \hat{\mathbf{u}} p - p \nabla \cdot \frac{1}{\rho a^2} \hat{\mathbf{u}} + \nabla \cdot \hat{\mathbf{u}} - \nabla \cdot \mathbf{A}^{-1} \nabla p = 0 \]  

(3.40)
The solution procedure is a modified PISO algorithm and is illustrated in figure 3.6. A detailed explanation of the PISO algorithm is given by Ferziger and Perić [40].

<table>
<thead>
<tr>
<th>Time loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for density (3.2)</td>
</tr>
<tr>
<td>Calculate liquid and gas densities using equations (3.32) (3.31)</td>
</tr>
<tr>
<td>Calculate void fraction (3.33)</td>
</tr>
<tr>
<td>Calculate sonic velocity (3.30)</td>
</tr>
<tr>
<td>Calculate viscosity (3.34)</td>
</tr>
<tr>
<td>Calculate velocity predictor (3.38)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PISO loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate velocity without contribution of pressure (3.38)</td>
</tr>
<tr>
<td>Solve for pressure (3.40)</td>
</tr>
<tr>
<td>Correct velocity (3.37)</td>
</tr>
</tbody>
</table>

Figure 3.6: Solution procedure

### 3.6.2 Solution Procedure - Dowson Density

The isentropic cavitation model introduced above needs to be adapted to use the non-linear density-pressure relationship of equation 2.27 commonly used for modelling EHL contacts. The definition of the sonic velocity $a$ in general is

$$ a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} $$

(3.41)

with $\frac{\partial p}{\partial \rho}$ as an isentropic change. The pressure-density relationship by Dowson 2.27 was given by

$$ \rho(p) = \rho_0 \frac{5.9 \cdot 10^8 + 1.34p}{5.9 \cdot 10^8 + p}, $$

(3.42)

Applying equation 3.41 to equation 2.27 gives the liquid sonic velocity as

$$ a_{l,dow} = \sqrt{\frac{(5.9 \cdot 10^8 + p)^2}{5.9 \cdot 10^8(1.34 - 1)}} \frac{1}{\rho_0}, $$

(3.43)
The constant sonic velocity of the liquid \(a_l\) in equation 3.30 is then replaced by the Dowson sonic velocity of equation 3.43.

### 3.6.3 Stability

The isentropic cavitation model outlined above is only stable if the sonic velocity of vapour and the saturated density of vapour are chosen within a certain range.

#### Sonic Velocity

If the sonic velocity of vapour \(a_v\) is set below a certain limit and when the maximum vapour fraction in the flow field approaches unity, a solution singularity in the pressure equation occurs. In this study a criterion was found which can predict the stable range of the velocity of vapour. The criterion was motivated by equation 3.29, which may be restated here as

\[
a_v^2 \frac{D \rho}{Dt} = \frac{D p}{Dt} \tag{3.44}
\]

When reaching unity vapour fraction, the sonic velocity in the cavitating region is equal to the vapour sonic velocity. Then, with equation 3.44, \(\Delta p\) as the maximum pressure range of a case and \(\Delta \rho\) as the maximum density range of a case, the minimum sonic velocity of vapour \(a_{v,min}\), above which a computation is stable, may be expressed as

\[
a_{v,min} = \sqrt{\frac{\Delta p}{\Delta \rho}} \tag{3.45}
\]

For example, in a case of \(\rho_l = 850 \text{ kg/m}^3\) the maximum density difference possible is \(\Delta \rho = \rho_l = 850 \text{ kg/m}^3\). If the maximum pressure occurring in this example is \(p_{\text{max}} = 3.5 \cdot 10^7 \text{ Pa}\) and the pressure in the cavitation region is \(p_{\text{vapour}} = 5000 \text{ Pa}\) then the maximum pressure difference is roughly \(\Delta p = p_{\text{max}} = 3.5 \cdot 10^7 \text{ Pa}\). Then, using equation 3.45 gives in that case.

\[
a_{v,min} = 203 \text{ m/s}
\]

If the sonic velocity of vapour is set below that limit a solution singularity occurs when the maximum vapour fraction in the flow field approaches unity.

This criterion matches well the stability limit experienced in many previous calculations. As another example, if the maximum pressure difference is \(\Delta p =\)
0.5 \cdot 10^9$, with a maximum density difference of $\Delta \rho = 850 \text{ kg/m}^3$, than the minimum vapour sonic velocity is about $a_{v, \text{min}} = 767 \text{ m/s}$.

**Density**

If the saturated density of vapour $\rho_{v, \text{sat}}$ is set below a certain limit, a solution singularity in the pressure equation occurs. This is accompanied by having Mach numbers $Ma = u/a$ approaching unity in the cavitation region. In an EHL contact the velocities of the surfaces would not usually be more than $u = 10 \text{ m/s}$, which would usually not indicate high Mach numbers. But, the mixture sonic velocity model given by Wallis in equation 3.30 results in mixture sonic velocities down to $a = 3 \text{ m/s}$. In this study a criterion was found which can predict the stable range of the saturated density of vapour by avoiding high Mach numbers in the flow field. It was found that a maximum Mach number below $Ma = 0.5$ would ensure stability. A criterion for the lower limit of the saturated density of vapour $\rho_{v, \text{sat}}, \text{min}$ can be found by looking at the relation for the mixture sonic velocity of equation 3.30. This equation can simplified with $\rho_l >> \rho_v$ to

$$
\frac{1}{a^2} \approx \alpha (1 - \alpha) \frac{\rho_l}{\rho_v a_v^2}
$$

(3.46)

The minimum sonic velocity and hence maximum Mach number, according to equation 3.30 occurs at vapour fraction $\alpha = 0.5$. With $a$ replaced by $a_{\min}$ as the minimum allowable sonic velocity, the minimum saturated density of vapour is given by

$$
\rho_{v, \text{sat}, \text{min}} = \frac{1}{4} \rho_{l, \text{sat}} \frac{a_{\text{min}}^2}{a_v^2}
$$

(3.47)

$a_{\min}$ may be estimated with

$$
a_{\min} = \frac{u_{\max}}{Ma_{\max}}
$$

(3.48)

where $u_{\max}$ is the maximum velocity occurring in the flow field, typically determined by the velocity of the bounding walls, and $Ma_{\max} = 0.5$ may be the stable upper limit of the Mach number.

For example, in a case with a moving wall velocity of $u_{\max} = 1 \text{ m/s}$ and $\rho_{l, \text{sat}} = 850 \text{ kg/m}^3$, the lower limit of the vapour sonic velocity would be

$$
\rho_{v, \text{sat}, \text{min}} = 0.0168 \text{ kg/m}^3
$$
The derived criterion for density matches the experienced stability limit well in other calculations, but it is not as sharp as the criterion for the sonic velocity.

### 3.6.4 New Model for Mixture Sonic Velocity

The Wallis model for mixture sonic velocity is only accurate for a vapour fraction up to 0.1. A more recent model has been developed by Chung [63] for bubbly flow and he has validated this against experiments for air-water systems. According to Chung the agreement with experiments is good up to a vapour fraction of 0.3. His formula is

\[
a = \frac{a_v a_l \sqrt{\frac{\rho_v a_v^2}{(1-\alpha)\rho_v a_v^2 + \alpha \rho_l a_l^2}}}{(1-\alpha) a_v + \alpha a_l \sqrt{\frac{\rho_v a_v^2}{(1-\alpha)\rho_v a_v^2 + \alpha \rho_l a_l^2}}} \]  (3.49)

Figure 3.7 compares Wallis and Chung models for various values of \( \rho_l \) and \( \rho_v \). The Chung model results in higher minimum sonic velocities and the minima occur at lower vapour fractions.

![Figure 3.7: Comparison of models for sonic velocity](image)

### 3.7 Isobaric Cavitation Model

The cavitation model introduced in this chapter, developed by Weller [61], will be named isobaric cavitation model as it tries to maintain the specified cavitation
pressure \( p_{\text{sat}} \) inside the cavitation region. If pressure drops below \( p_{\text{sat}} \), liquid is converted into vapour. If there is no liquid to convert, the pressure will drop below cavitation pressure. If the pressure rises above \( p_{\text{sat}} \) vapour is converted into liquid. With those two mechanisms the pressure in a cavitating region is kept close to the cavitation pressure. It is assumed that vapour and liquid are homogenous phases in thermodynamic equilibrium. The density of vapour is given by

\[
\rho_v = \psi_v p
\]  

(3.50)

where \( \psi_v \) is the compressibility of vapour. The density of the liquid phase is given by

\[
\rho_l = \rho_{l,0} + \psi_l p
\]  

(3.51)

where \( \rho_{l,0} \) is the liquid density at zero pressure and is \( \psi_l \) is the compressibility of liquid. The vapour fraction is defined as

\[
\alpha = \frac{\rho - \rho_{l,\text{sat}}}{\rho_{v,\text{sat}} - \rho_{l,\text{sat}}}
\]  

(3.52)

where \( \rho_{l,\text{sat}} \) is the liquid density and \( \rho_{v,\text{sat}} \) is the vapour density at \( p_{\text{sat}} \). The compressibility of the vapour/liquid mixture \( \psi \) can be chosen from three different models. The simplest is the linear interpolation model:

\[
\psi = \alpha \psi_v + (1 - \alpha) \psi_l
\]  

(3.53)

The Wallis and the Chung models for bubbly flows as outlined in section 3.6 are available as well. The mixture density is given by

\[
\rho = \psi (p - p_{\text{sat}}) + (1 - \alpha) \rho_{l,0} + (\alpha \psi_v + (1 - \alpha) \psi_l) p_{\text{sat}}
\]  

(3.54)

This is a general function of density which can used with all three compressibility models available. It is set up such that the state at \( p = p_{\text{sat}} \) serves as the reference point. In the case of the linear compressibility mixing model, equation 3.54 becomes

\[
\rho = \alpha \psi_v p + (1 - \alpha) (\rho_{l,0} + \psi_l p)
\]  

(3.55)

The mixture dynamic viscosity is assumed to be:

\[
\mu = \alpha \mu_v + (1 - \alpha) \mu_l
\]  

(3.56)
Alternatively a logarithmic mixing law taken from Reid [64], given below, has been tested in this study. It had a too negative effect on stability and could not be used. It might be possible to stabilise it, but no further investigation was attempted.

\[ \ln \mu = \alpha \ln \mu_v + (1 - \alpha) \ln \mu_l \]  \hspace{1cm} (3.57)

The conservation of mass is given by equation [3.2] the conservation of momentum is given by equation [3.3] assuming both phases travelling at the same velocity.

### 3.7.1 Solution Procedure

The discretised Laplacian equation for calculating the pressure field is derived as follows. The mixture density equation [3.54] is inserted into the continuity equation [3.2] gives:

\[
\frac{\partial (\psi_p)}{\partial t} + \frac{\partial \alpha}{\partial t} ((\psi_v - \psi_l) p_{sat} - \rho_l,0) - \frac{\partial \psi}{\partial t} p_{sat} + \nabla \cdot (\rho u) = 0 \]  \hspace{1cm} (3.58)

The discretised momentum equation can be expressed in the form

\[
A \cdot u = H(u) - \nabla p \]  \hspace{1cm} (3.59)

where \( A \) contains the diagonal matrix coefficients and \( H(u) \) contains all off-diagonal matrix coefficients multiplied by their corresponding velocities. With equation [3.59] and \( p^{t-1} \) as the pressure of the previous timestep the velocity predictor \( \tilde{u} \) is defined as

\[
\tilde{u} = A^{-1} H - A^{-1} \nabla p^{t-1} \]  \hspace{1cm} (3.60)

By using equation [3.60] the term \( \nabla \cdot (\rho u) \) in equation [3.58] can be expressed as

\[
\nabla \cdot (\rho u) = \nabla \cdot (\rho \tilde{u}) + \nabla \cdot (\rho A^{-1} \nabla p^{t-1}) - \nabla \cdot (\rho A^{-1} \nabla p) \]  \hspace{1cm} (3.61)

Inserting equation [3.61] into equation [3.58] gives the final form of the pressure equation:

\[
\frac{\partial (\psi_p)}{\partial t} + \frac{\partial \alpha}{\partial t} ((\psi_v - \psi_l) p_{sat} - \rho_l,0) - \frac{\partial \psi}{\partial t} p_{sat} \\
+ \nabla \cdot (\rho \tilde{u}) + \nabla \cdot (\rho A^{-1} \nabla p^{t-1}) - \nabla \cdot (\rho A^{-1} \nabla p) = 0 \]  \hspace{1cm} (3.62)
3.7.2 Solution Procedure - Dowson Density

The isobaric cavitation model introduced above needs to be adapted to use the non-linear density-pressure relationship of equation 2.27 commonly used for modelling EHL. The sonic velocity of a Dowson liquid was defined by equation 3.43. The liquid compressibility is given by

$$\psi_{l,dow} = \frac{1}{a_{l,dow}^2}$$ \hspace{1cm} (3.63)

The mixture density in general is

$$\rho = \alpha \rho_v + (1 - \alpha) \rho_l$$ \hspace{1cm} (3.64)

Temporal differentiation of equation 3.64 gives

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t} (\alpha \rho_v + (1 - \alpha) \rho_l)$$ \hspace{1cm} (3.65)

This can be expanded as follows

$$\frac{\partial \rho}{\partial t} = \frac{\partial \alpha}{\partial t} (\psi_v \rho_v - \rho_l) + \alpha \frac{\partial \rho_v}{\partial p} \frac{\partial \rho}{\partial p} + (1 - \alpha) \frac{\partial \rho_l}{\partial p} \frac{\partial \rho}{\partial t}$$ \hspace{1cm} (3.66)

Inserting $$\rho_v = \psi_v p$$, $$\rho_l = \rho_{l,dow}$$ and $$\psi_{l,dow}$$ into the equation above gives

$$\frac{\partial \rho}{\partial t} = \frac{\partial \alpha}{\partial t} (\psi_v p - \rho_{l,dow}) + \alpha \psi_v \frac{\partial \rho}{\partial t} + (1 - \alpha) \psi_{l,dow} \frac{\partial \rho}{\partial t}$$ \hspace{1cm} (3.67)

The convective term in the continuity equation is expressed as in equation 3.61 and $$\frac{\partial \rho}{\partial t}$$ by equation 3.67, resulting in the following:

$$\frac{\partial \alpha}{\partial t} (\psi_v p^{t-1} - \rho_{l,dow}(p^{t-1})) + \alpha \psi_v \frac{\partial \rho}{\partial t} + (1 - \alpha) \psi_{l,dow}(p^{t-1}) \frac{\partial \rho}{\partial t} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \dot{\mathbf{u}}) + \nabla \cdot (\rho \mathbf{A}^{-1} \nabla p^{t-1}) - \nabla \cdot (\rho \mathbf{A}^{-1} \nabla p) = 0$$ \hspace{1cm} (3.68)

3.7.3 Temperature Equation

In this section the temperature equation for the isobaric cavitation model is derived. The only way temperature feeds back into the fluid solution is via the temperature dependency of the viscosity. In an EHL contact viscous heating and conduction are the dominant terms in the energy balance [2], which resulted in the decision to
develop an energy equation in terms of temperature, as the conduction term can be modelled implicitly.

**Phasic Enthalpies**

The total differential for enthalpy of a fluid is

\[
dh = \left( \frac{\partial h}{\partial T} \right)_p dT + \left( \frac{\partial h}{\partial p} \right)_T dp
\]  
(3.69)

From the Maxwell equations with \( v \) as the specific volume

\[
\left( \frac{\partial h}{\partial p} \right)_T - v = -T \left( \frac{\partial v}{\partial T} \right)_p
\]  
(3.70)

and with \( C_p = \left( \frac{\partial h}{\partial T} \right)_p \), equation 3.70 becomes

\[
dh = C_p dT + \left[ v - T \left( \frac{\partial v}{\partial T} \right)_p \right] dp
\]  
(3.71)

The equation of state for an ideal gas is

\[
v = \frac{RT}{p}
\]  
(3.72)

With equation 3.72 equation 3.71 becomes

\[
dh = C_{p,v}dT
\]  
(3.73)

which, when integrated for the vapour phase gives

\[
h_v = h_{v,0} + C_{p,v}(T - T_0)
\]  
(3.74)

The enthalpy of or a compressible liquid can be expressed by inserting \( \rho_{l,dow} \) from equation 2.27 into equation 3.71 and assuming density being constant with temperature, leads to

\[
dh = C_{p,l}dT + \frac{dp}{\rho_{l,dow}}
\]  
(3.75)
and in integral form

\[ h_l = h_{0,l} + C_{p,l} (T - T_0) + \int_{0}^{p} \left[ \frac{1}{\rho_{l,doc}} \right] d\hat{p} \]  \hspace{1cm} (3.76)

With \( D_a = 5.9 \cdot 10^8 Pa \) and \( D_b = 1.34 \), which are the coefficients of the Dowson liquid of equation 2.27, equation 3.76 can be integrated, leading to

\[ h_l = h_{0,l} + C_{p,l} (T - T_0) + \frac{D_b p + D_a (D_b - 1) \ln(1 + \frac{D_b}{D_a} p)}{D_b^2 \rho_{l,0}} \]  \hspace{1cm} (3.77)

**Temperature Equation for an Ideal Gas**

The term \( \frac{D(\rho h_v)}{Dt} \) in the energy equation 3.5 can be expanded to

\[ \frac{D(\rho h_v)}{Dt} = \rho \frac{Dh_v}{Dt} + h_v \frac{D\rho}{Dt} = \rho \frac{Dh}{Dt} \]  \hspace{1cm} (3.78)

where \( \frac{D\rho}{Dt} \) is the continuity equation 3.2 and is equal to zero. Insertion of equation 3.73 gives

\[ \rho_v \frac{Dh_v}{Dt} = \rho_v \left( C_{p,v} \frac{DT}{Dt} \right) \]  \hspace{1cm} (3.79)

This, when inserted into equation 3.5, results in the final equation for the temperature for an ideal gas

\[ \rho_v C_{p,v} \frac{DT}{Dt} = \nabla \cdot k \nabla T - \tau : \nabla U + \frac{Dp}{Dt} \]  \hspace{1cm} (3.80)

**Temperature Equation for a Compressible Liquid**

Insertion of equation 3.75 into 3.78 gives

\[ \rho_l \frac{Dh}{Dt} = \rho_l \left( C_{p,l} \frac{DT}{Dt} + \frac{1}{\rho_l} \frac{Dp}{Dt} \right) \]  \hspace{1cm} (3.81)

which can be inserted into equation 3.5, the \( \frac{Dp}{Dt} \) terms cancelling out, and results in

\[ \rho_l C_{p,l} \frac{DT}{Dt} = \nabla \cdot k \nabla T - \tau : \nabla U \]  \hspace{1cm} (3.82)
Mixture Temperature Equation

It is assumed that liquid phase and vapour phase share the same pressure and temperature locally.

The mixture energy is

\[ \rho h = \alpha \rho_v h_v + (1 - \alpha) \rho_l h_l \]  

which leads to

\[ d(\rho h) = d[\alpha \rho_v h_v] + d[(1 - \alpha) \rho_l h_l] \]

Expanding and taking the substantial derivative gives

\[ \frac{D(\rho h)}{Dt} = h_v \frac{D(\alpha \rho_v)}{Dt} + \alpha \rho_v \frac{D(h_v)}{Dt} \]

\[ + h_l \frac{D((1 - \alpha) \rho_l)}{Dt} + (1 - \alpha) \rho_l \frac{D(h_l)}{Dt} \]

In this, the phasic enthalpies 3.73 and 3.75 are inserted and assuming \( \frac{D\rho_v}{Dt} = \frac{D\rho_l}{Dt} = 0 \), which is justified since those densities are only a function of pressure and, in the cavitating region, the pressure-level remains very close to the cavitation pressure, this leads to

\[ \rho \frac{D(\rho h)}{Dt} = h_v \rho_v \frac{D(\alpha)}{Dt} + \alpha \rho_v C_{p,v} \frac{D(T)}{Dt} \]

\[ - h_l \rho_l \frac{D(\alpha)}{Dt} + (1 - \alpha) \rho_l \left[ C_{p,l} \frac{D(T)}{Dt} + \frac{1}{\rho_l} \frac{Dp}{Dt} \right] \]

This, compared to equation 3.5, gives the final form of the mixture temperature equation

\[ \alpha \rho_v C_{p,v} \frac{D(T)}{Dt} + (1 - \alpha) \rho_l \left[ C_{p,l} \frac{D(T)}{Dt} \right] + \frac{D(\alpha)}{Dt} (\rho_v h_v - \rho_l h_l) \]

\[ = \nabla \cdot k \nabla T - \tau : \nabla U + \alpha \frac{Dp}{Dt} \]

Equation 3.87 can be seen as an interpolation between the temperature equation for an ideal gas 3.80 and the temperature equation for a compressible liquid 3.82, with an additional term for phase change.
3.8 Comparison of Cavitation Models

In this section the two cavitation models introduced in sections 3.6 and 3.7 are applied to a test case and the results are compared and discussed in order to examine the behaviour of both models.

3.8.1 Test Case

The domain consists of a half-cylinder in 2D below a flat surface as shown in figure 3.1. The flat surface is moving at $1 \text{ m/s}$, which is moderate velocity for an EHL contact. The operating parameters are summarised in table 3.1; the viscosity is that of a typical lubricant in a bearing. The viscosity is not yet piezo-viscous at the resulting pressure level.

The grid used is based on the 2/2 refined grid of section 3.4. Table 3.3 shows the parameters of the reference case of this study. Figure 3.8 shows the domain and boundary conditions. Fluid parameter values are chosen which reflect a typical mineral oil at $T = 353.15K$. The molecular weight $M = 0.282 \frac{kg}{mol}$ is that of $n$-eicosane ($C_{20}H_{42}$). With the universal gas constant $R = 8.3144 \frac{J}{molK}$, and the vapour pressure $p_{\text{vapour}}$, the vapour density $\rho_{v,sat}$ is calculated using the ideal gas law.

$$\rho_{v,sat} = \frac{p_{vapour}}{\frac{RT}{M}}$$

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder radius</td>
<td>$R = 10mm$</td>
</tr>
<tr>
<td>length of domain</td>
<td>$D = 120mm$</td>
</tr>
<tr>
<td>surface velocity</td>
<td>$v = 1 \text{ m/s}$</td>
</tr>
<tr>
<td>film thickness</td>
<td>$h_0 = 10^{-7} m$</td>
</tr>
<tr>
<td>dynamic viscosity liquid</td>
<td>$\mu_l = 10^{-2} \frac{m^2}{s}$</td>
</tr>
<tr>
<td>dynamic viscosity vapour</td>
<td>$\mu_v = 8.97 \times 10^{-6} \frac{m^2}{s}$</td>
</tr>
<tr>
<td>vapour pressure</td>
<td>$p_{vapour} = 3000 \text{ Pa}$</td>
</tr>
<tr>
<td>environment pressure</td>
<td>$p_0 = 10^5 \text{ Pa}$</td>
</tr>
<tr>
<td>liquid density at $p_0$</td>
<td>$\rho_{l,sat} = 850 \frac{kg}{m^3}$</td>
</tr>
<tr>
<td>vapour density at $p_{vapour}$</td>
<td>$\rho_{v,sat} = 0.288 \frac{kg}{m^3}$</td>
</tr>
<tr>
<td>sonic velocity vapour</td>
<td>$a_v = 225 \text{ m/s}$</td>
</tr>
<tr>
<td>sonic velocity liquid</td>
<td>$a_l = 1200 \text{ m/s}$</td>
</tr>
</tbody>
</table>

Table 3.3: reference case - parameters
**Boundary conditions**

- **Density**
  - zero gradient (all boundaries)
- **Pressure**
  - zero gradient (moving Wall, fixed Wall)
  - constant $10^{-5}$ pa (inlet, outlet)
- **Velocity**
  - 1 m/s in x direction (moving wall)
  - zero (fixed wall)
  - calculated from flux, normal to patch (inlet, outlet)

![Figure 3.8: Domain and boundary conditions](image)

3.8.2 Numerical Parameters

The test cases are run until a steady-state solution is obtained. The solution algorithm used for solving the pressure equation and the continuity equation is the *Incomplete-Cholesky preconditioned bi-conjugate gradient* method. For all divergence terms the *upwind* (first order) discretisation scheme was used for stability reasons. All gradient terms are evaluated using a *Gauss linear* discretisation (second order). The Laplacian terms in the pressure and momentum equation are evaluated using *Gauss linear corrected*, which is second order accurate, unbounded and conservative.

3.8.3 Results

Five models are studied, the isentropic cavitation model with the Chung and Wallis compressibility models and the isobaric cavitation model with the linear, Chung and Wallis compressibility models. Figure 3.9 shows the predicted pressure distribution. Significant pressure variations occur only near the origin where the fluid film is the thinnest. The pressure rises up to about $34MPa$ and drops to small
or negative values in the cavitation region. The Chung and Wallis compressibility models result in slightly different pressure distributions when applied to the isentropic model. For the isobaric model there is virtually no difference in pressure distribution between linear, Chung and Wallis compressibility model.

Figures 3.10 and 3.11 show pressure and vapour fraction in the cavitating region for the isentropic Model with Chung and Wallis compressibility. Figures 3.12, 3.13 and 3.14 show pressure and vapour fraction in the cavitating region for the isobaric Model with Chung, Wallis and linear compressibility. The differences between the compressibility models for the isobaric model are very small, the pressure in the cavitation region is very close the the specified cavitation pressure of 3000 Pa. The Chung and Wallis models are more stable than the linear compressibility, which required a timestep of $10^{-8}$ s for stability instead of $10^{-7}$ s.

Both models predict a cavitation region which starts near the origin and extents downstream to up to $x = 0.4 \text{ mm}$. The isobaric model predicts a sharp end of the cavitation region with a pressure whereas the isentropic model predicts more complex cavitation region and differences between compressibility models are significant. The minimum pressure for the Chung model is $1.5 \times 10^{-5} \text{ Pa}$, for the Wallis
model the minimum pressure is $0.3 \cdot 10^{-5} Pa$. These pressure differences cause different pressure distributions for the entire contact as shown in figure 3.9. Negative fluid pressure have been reported [50, 51, 52] in piston rings and journal bearings up to $-7.8 \cdot 10^{-5} Pa$. The negative pressures predicted by the isentropic model are therefore in the realms of possibility, but with the underlying assumption of isentropic, non-viscous flows the value of such predictions is uncertain.

3.9 Closure

In this chapter the governing equation for fluid flow and the finite volume method have been introduced. A grid study was performed, pointing to the importance of appropriate aspect ratios of the computational cells. The isentropic and the isobaric cavitation model have been introduced and compared. Both models have been extended to use the Dowson pressure-density relationship.

The isentropic model was originally developed for isothermal, non-viscous flows. Nonetheless, it can operate in highly viscous conditions and can produce reasonable results. It suffers from non-physical restrictions, required for stability reasons, on vapour sonic velocity and vapour density. The pressure in the cavitation region can assume negative pressures, which has been shown by experiments on journal
Figure 3.11: Isentropic cavitation - Wallis compressibility

Figure 3.12: Isobaric cavitation - Chung compressibility
Figure 3.13: Isobaric cavitation - Wallis compressibility

Figure 3.14: Isobaric cavitation - linear compressibility
bearings, but the validity of those predictions is uncertain, as this model has been developed for isentropic flows and not for highly-viscous flows.

The isobaric cavitation model maintains the cavitation pressure specified for steady-state in the cavitating region. It does not pose any unphysical restrictions on the fluid properties and is stable for density ratios up to 1:10000. It can be used for isothermal or thermal and viscous and non-viscous flow regimes. An energy equation has been developed for it. Negative pressures cannot be modeled inside the cavitation region for steady-state with the isobaric cavitation model, but it might be possible to extend the model in that direction.
Chapter 4

Elastic Deflection in CFD

In this chapter the treatment of the deforming solid bodies following the Hertzian contact theory is introduced and validated against analytical solutions. The deflection algorithm implemented is suitable for two-dimensional and three-dimensional calculations. It is based on equation 2.5 introduced in section 2.2.3. Generally, the calculation of the deflection $w_p$ at a given point is reduced to a simple summation for two and three dimensions, where $g_{p,i}$ is the influence coefficient and $p_i$ is the local pressure.

$$w_p = \sum_{\text{all } i} g_{p,i} p_i$$ (4.1)

4.1 General Procedure

The deflection algorithm treats surface points in two different ways:

1. Only the surface points are moved.

2. The surface points are moved and that movement causes deformation of the internal mesh connected to them.

The first option can be applied far away from the contact, where the deflection can be accommodated within the first cell layer. This part of the algorithm works with any cell shape.

The second option needs to be applied in the central part of the contact, because the surface deflections can be much bigger than the actual film thickness. Hence, the first cell layer can not accommodate the deflection changes. This part of the algorithm rests on the assumptions, that

- the deforming mesh consists of hexahedrons,
- the deforming mesh is block-structured
- the mesh is uniform in the direction of deformation

Figure 4.1 shows the different approaches conceptually. The central part of the mesh is deformed according to the surface deformation, whereas in the outer part only the surface points are moved. The fixed reference point is indicated as a blue dot. The surface points only move perpendicular to the non-deforming wall according to equation 2.3. The position of point $i$ is calculated relative to the fixed reference point, leading to the film thickness

$$h_i = h_{c0} + h_u - (w_i - w_{ref})$$

where $h_{c0}$ is the undeformed film thickness at the origin, $h_u$ is the undeformed geometry, $w_i$ and $w_{ref}$ are the deflection according to equation 4.1 of the surface point $i$ and the reference point.

![Figure 4.1: Deforming mesh](image)

The deformation of the internal mesh is illustrated in figure 4.2. $\vec{w}_i$ denotes the deflection of the surface point according to equation 4.2. $m$ is the number of mesh points connected to the surface point and $j$ describes how many points a mesh point is away from its corresponding surface point. With $r_j^*\vec{r}$ as the initial position...
the new position is

\[ r_{j,\text{def}} = r_j + w_t \cdot \left( 1.0 - \frac{j}{m - 1} \right) \] (4.3)

Mesh motion according to equation 4.3 is very fast and the change of volume of one cell depends only on four surface points. This might be exploited in future to have a more implicit coupling between pressure and deflection.

### 4.2 Parallelisation

To tackle the problem of elastohydrodynamic lubrication using a CFD approach is very expensive. Especially in three dimensions the time needed to perform such calculations is too much on a single processor. Therefore a parallelisation procedure is implemented, which works for both 2D and 3D cases to speed up the calculations.

When preparing a case for a parallel run, the domain is decomposed into smaller sections, which are then assigned to different processors. The decomposition is done
automatically and aims for equal calculation time for the fluid flow. The deflection calculation needs to work with this already decomposed case and aims for an equal calculation load as well. Equation 2.7 is a simple summation and can be parallelised by distributing the moving surface points equally among all processors. With that strategy the decomposition of the fluid part and the decomposition of the deflection part are completely independent and this enables the equal use of all processors. But, as can be seen from equation 2.7, the movement of one mesh point depends on the pressure of all faces in the Hertzian contact. That leads to the need of communicating some information between the processors before and during the parallel deflection calculation.

An example for such a decomposition with four processors is illustrated in figure 4.3. Processor0 in this example is not in contact with the deforming surface, but is responsible for calculating the position of three surface points, as indicated by the colouring. In order to do that, the position of those surface points and the pressure of all surface-faces has to be communicated from all processors holding surface-faces to Processor0. With that information the new position of those points can be calculated by Processor0 and then communicated back to all other processors.

An additional complication is, that in the centre of the contact, a point’s position depends on the position of the surface-point above it, as described by equation 4.3. But in the general case, those surface-points and their connected internal points are on different processors. So, before starting a calculation, the order of the internal points and their corresponding surface point has to be established. With that order and the new position of the surface points, the new position of the internal points can be calculated.

![Figure 4.3: Parallelisation of deflection calculation](image)
4.3 2D implementation

Most parts of the deflection algorithm are independent of whether the problem is two-dimensional or three-dimensional. In the two-dimensional case the geometry and the equation for the influence coefficients needs to be specified. Usually, in the traditional Reynolds approach a uniform mesh is used in order to store the influence coefficients and save computational time. However, in this work it was found that calculating the influence coefficients when required makes the deflection calculation at most as expensive as the fluid part. This enables the use of a non-uniform mesh, which is necessary to keep the computational costs for the fluid part down.

For validation, a load of \( L = 10^5 \text{Pa} \) is imposed with a reduced Youngs modulus of \( E_r = 3.4522996 \cdot 10^{11} \text{Pa} \) and \( b \) as the roller depth. For a two-dimensional case the roller depth \( b \) should be infinity, but that would lead to infinite deflection according to equation 4.5. As only deflection differences are employed the roller depth \( b \) can be set to any arbitrary non-zero value. The resulting contact radius is \( a = 0.0001076405 m \). Introducing the dimensionless deflection \( \bar{w} = \frac{\pi E_r b}{2L} \), dimensionless thickness \( \bar{b} = \frac{b}{a} \) and dimensionless x-coordinate \( \bar{x} = \frac{x}{a} \) and following Gohar [2], the dimensionless deflection inside the contact radius is

\[
\bar{w}_i = \pi \left( \ln \left( 4\bar{b} \right) + 0.5 - \bar{x}^2 \right) \tag{4.4}
\]

and the dimensionless deflection outside the contact radius is

\[
\bar{w}_o = \pi \left( \bar{x} \sqrt{\bar{x}^2 - 1} - \ln \left( x + \sqrt{x^2 - 1} \right) + (ln(4\bar{b}) + 0.5 - \bar{x}^2) \right) \tag{4.5}
\]

The same condition are imposed on the developed algorithm and compared to the analytical solution. The result are shown in figure 4.4. The relative error, normalised by the maximum deflection, is shown in figure 4.5. The maximum error occurs at the edge of the contact radius and is below 0.014 percent, which should be accurate enough.
Figure 4.4: 2D deflection validation

Figure 4.5: 2D deflection validation - relative error
4.4 3D implementation

In order to understand the three-dimensional deflection algorithm the mesh structure for the fluid calculation has to be introduced. The computational domain consists of a quarter-sphere, taking advantage of the symmetry-plane perpendicular to the flow and deflection direction. Therefore, each face is accounted for twice in equation 4.1. Firstly with its original coordinates and secondly with the coordinates mirrored at the symmetry-plane.

An O-grid topology is employed, which is suitable for meshing a sphere. Only the sphere is deforming and its vertical position is adjusted for load control. The flat surface is rigid and can be moving in the flow direction. Figure 4.6 shows the quarter-sphere, the symmetry-plane and the rigid wall of the three dimensional grid. Figures 4.7, 4.8, 4.9 show the O-grid topology of the rigid wall. In the centre of the contact the surface faces are uniform squares. In the surroundings, the faces are not regular, which is significant for the deflection calculation.

As for the 2D case, the basis for the 3D algorithm is equation 2.5 which originates from the Hertzian contact theory. That equation is evaluated assuming rectangular faces, which is the case in the high pressure, centre of contact region. The outer faces have irregular shapes, which are calculated as squares with the same centre and face area. This assumptions allows the use of a relatively simple equation for the deflection due to a pressured face. Equation 4.6 is taken from Gohar [2] with \( c \) and \( d \) as half-widths in the \( y \) and \( x \) direction.

\[
\begin{align*}
w_{p,i} &= \frac{p_i}{E_r} \left\{(y - d) \ln \left(\frac{(x - c) + \sqrt{(y - d)^2 + (x - c)^2}}{(x + c) + \sqrt{(y - d)^2 + (x + c)^2}}\right)
+ (y + d) \ln \left(\frac{(x + c) + \sqrt{(y + d)^2 + (x + c)^2}}{(x - c) + \sqrt{(y + d)^2 + (x - c)^2}}\right)
+ (x + c) \ln \left(\frac{(y + d) + \sqrt{(y + d)^2 + (x + c)^2}}{(y - d) + \sqrt{(y - d)^2 + (x + c)^2}}\right)
+ (x - c) \ln \left(\frac{(y - d) + \sqrt{(y - d)^2 + (x - c)^2}}{(y + d) + \sqrt{(y + d)^2 + (x - c)^2}}\right)\right\}
\end{align*}
\] (4.6)

With equation 4.6 the deflection calculation can reduced to a simple sum.
Figure 4.6: Three dimensional mesh
Figure 4.7: 3D mesh - rigid wall (1)
Figure 4.8: 3D mesh - rigid wall (2)
Figure 4.9: 3D mesh - rigid wall (3)
This approach applied to the grid shown \((4.6, 4.7, 4.8, 4.9)\) leads to very long computational times. On a single processor the time needed to calculate the fluid is 2-3 seconds while the time needed for the deflection is about 200 seconds, which is far too much for a realistic case. In order to bring this computational time down to the level of the fluid calculation, a more complicated approach is chosen. The principle is shown in figure 4.10, which depicts the topology of the deforming quarter-space.

In the centre of the contact the grid is uniform and is assumed that only that region is responsible for deforming the entire sphere, which is a fair assumption since the pressure level outside that region is so small that it has no a significant impact on the deformation. But this alone is not sufficient to reduce the computational time enough. The influence coefficients \(g_{p,i}\) of equation 4.1 still have to be calculated as needed and cannot be stored. If, for example, the pressured patch consists of 100 times 200 faces and the entire sphere has got twice as many nodes. Then, using double precision, which uses 8 byte of memory, we arrive at \(20,000 \times 40,000 \times 8\) Byte = 6.10 GB just for storing the influence coefficients.

In order to reduce the number of influence coefficients needed to be stored, the uniform grid of the centre is extended to cover the entire sphere for the deflection calculation as shown in figure 4.10. On such a grid an influence coefficient only depends on the face area \(S_f\) and the distance from face to deflected point \(\Delta x\) and \(\Delta y\). The face area \(S_f\) is constant and \(\Delta x\) and \(\Delta y\) assume only values a constant interval.

The position of a point in such a grid can be described by the indices \(I_p\) and \(J_p\) in the \(x\) and \(y\) directions. Equally, the position of a face in such a grid can be described by the indices \(I_f\) and \(J_f\) in the \(x\) and \(y\) directions. Now, the influence coefficients covering all possible combinations of \(\Delta x\) and \(\Delta y\) are stored into a matrix. When performing the deflection calculation the appropriate influence coefficient is retrieved by performing simple integer operations between \(I_p, I_f\) and \(J_p, J_f\). For the points of the uniform pressured patch, this approach is exact in the limits of the model, but all the other points generally do not coincide with the points of the extended uniform grid. For those points, the deflection of the nearest point on the extended uniform grid is taken. This seems rather inaccurate, but, as will be shown in the next paragraph, the error made is smaller than the error introduced due to the discretisation of the Hertzian integral. As a result, the calculation time dropped for the example grid from 200 seconds to 0.4 seconds, which is less than the time for the fluid calculation of 2-3 seconds.
4.4.1 Validation

Johnson [3] gave an analytical solution for a point contact following the Hertzian theory. With load $L$, radius $R$ and reduced Youngs modulus $E_r$ the contact radius $a_c$ is given by

$$a_c = R \left( \frac{3}{4} \pi \frac{L}{R^2 E_r} \right)^{1/3} \tag{4.7}$$

The resulting maximum pressure $p_{max}$ is calculated by

$$p_{max} = 2 a_c \frac{E_r}{\pi^2 R} \tag{4.8}$$

With $r$ as the distance from the centre of the contact, the deformation $w_{H,i}$ inside the contact radius reads

$$w_{H,i} = \frac{\pi^2 p_{max}}{E_r 4 a_c} \left( 2a_c^2 - r^2 \right) \tag{4.9}$$

The deformation $w_{H,o}$ outside the contact radius reads

$$w_{H,o} = \frac{\pi p_{max}}{2 a_c E_r} \left[ \left( 2a_c^2 - r^2 \right) \sin^{-1}(a_c/r) + a_c r \left( 1 - \frac{a_c^2}{r^2} \right)^{0.5} \right] \tag{4.10}$$

Figure 4.10: Grid topology deforming of quarter-space
Outside the contact radius the pressure is zero, inside the resulting pressure distribution is

\[ p_H = p_{\text{max}} \frac{\sqrt{a_c^2 - r^2}}{a_c} \]  (4.11)

A typical test-case is applied to the deflection algorithm and compared to the analytical solution. The load applied is \( L = 20N \), radius \( R = 0.01m \) and the reduced Youngs modulus, glass on metal, is \( E_r = 1.9006011 \cdot 10^{11} \). That results in a contact radius of \( a = 0.0001353mm \) and the maximum pressure of \( p_{\text{max}} = 5.2128 \cdot 10^8Pa \). In this case the mesh spacing of the uniform patch is \( 1 \cdot 10^{-5} \), so that 27 faces cover the diameter of the contact. Across a face the pressure is assumed to be constant and the value is set to the analytical pressure at the face centre. With \( w_H \) as the analytical deflection, \( w_{H,n} \) as the numerical deflection and \( w_{H,max} \) as the maximum analytical deflection, the relative error is defined as

\[ \epsilon_{\text{rel}} = \frac{\|w_H - w_{H,n}\|}{w_{H,max}} \]  (4.12)

This approximation of the analytical pressure distribution is most inaccurate in regions of high pressure gradient. Hence, the biggest error occur at the fringes of the contact radius as shown in figure 4.11. It is important to note, that in the non-uniform part, where the grid points were snapped to the next point on the deflection-grid, the error is smaller than the discretisation error of the pressure.

### 4.5 Closure

In this chapter an algorithm developed was developed to deform the computational domain of the CFD following the Hertzian contact theory. It can be applied to two- and three-dimensional meshes and it has been validated against analytical solutions. The algorithm is parallelised and the computational time needed is less or equal than the time needed to solve for the fluid flow.
Figure 4.11: 3D deflection - relative error
Chapter 5

CFD Modelling of EHL

5.1 Introduction

In this chapter the cavitation models outlined in chapter 3 are coupled with the elastic deflection outlined in chapter 4 in order to obtain solutions for elastohydrodynamic systems. The applied fluid properties are introduced. The coupling of the fluid flow to the deflection algorithm is described. Steady-state results are presented using both cavitation models in two and three dimensions. The isentropic model is only used under isothermal conditions. With the isobaric cavitation model thermal effects are studied and compared to isothermal calculations.

5.2 Coupling of Fluid Flow and Deflection

5.2.1 Explicit Fluid/Deflection Coupling

In chapter 3 the cavitation treatment using OpenFOAM was outlined and the deflection calculation was explained in chapter 4. These two parts are integrated in a forward iterative manner modifying the solution procedure explained in section 3.6.1. At the beginning of each time step the new geometry is calculated as a function of pressure. The mesh is deformed as outlined in section 4. In transient calculations the velocity $u$ needed to be corrected with the velocity of the moving mesh. This is not done in this study as only steady-state solution are considered and not correcting the velocity for mesh motion is beneficial for the stability of the calculations.
5.2.2 Start-up Procedure and Load Control

Elastohydrodynamic lubrication in a metal-on-metal or metal-on-glass contact is a very delicate system. The deformation of the radius is in the range of 100 to 1000 times larger than the film thickness. The highly non-linear behaviour of viscosity adds to the difficulty. On top of this, cavitation modelling is known to be very unstable as well, which is caused by the big density gradients in the cavitating region. Delannoy [56] experienced problems with density ratios higher than 1 : 100, but realistic ones are up to 1 : 10000.

In a Reynolds-based approach the dry Hertzian contact pressure distribution is often taken as a starting point and with a gap according to some film thickness formula [2]. In this thesis this could not be achieved, the cavitation algorithm became unstable, even with very low under-relaxation factors for deflection.

In order to start with a valid solution for the fluid part, an initial gap was left between the two bodies, 0.1 µm for a radius of 0.01 m was found to be appropriate. From there, the bodies were slowly pressed together. If that was done too fast the two bodies collided or the fluid part became unstable.

The term pressed together needs explanation. As outlined in Chapter 4, the deformation is calculated in relation to a reference point. The reference point is not moved according to the Hertzian contact theory but its coordinates are controlled in the absolute reference frame. That is done by modifying the undeformed gap between the two bodies $h_0$ in equation 4.2, which is actually negative in an ehl-contact. Implemented are two approaches, fixed geometry and fixed load. During the start-up phase the velocity of the reference point is limited for both approaches. The maximum allowable velocity depends on the type parameters of an ehl-contact, the timestep, the deflection under-relaxation and the current pressure level. Generally one would start with a rather big timestep and high velocity of the reference point and let it go until the bodies collided are the fluid solution becomes unstable. Then reduce timestep and/or the velocity of the reference point until the desired pressure level is reached. That procedure requires experience and in this study it was not possible to come up with a general method for the start-up phase.
**Fixed Geometry**

The undeformed gap is expressed as

\[
h_{c0} = \begin{cases} 
h_{0,\text{start}} - (h_{0,\text{start}} - h_{0,\text{end}}) \frac{t}{t_{\text{start-up}}} & \text{for } t < t_{\text{start-up}} \\
h_{0,\text{end}} & \text{for } t \geq t_{\text{start-up}} \end{cases}
\]  

(5.1)

with \(t_{\text{start-up}}\) as the end of start-up period, \(h_{0,\text{end}}\) as the final gap and \(h_{0,\text{start}}\) as the initial gap.

**Fixed Load**

In experiments the applied load is usually prescribed, rather than a certain geometry. To mimic that in the fixed load approach the target load \(L_{\text{aim}}\) and a maximum speed of the reference point \(u_{h,\text{max}}\) is specified, each timestep the current load \(L\) is compared to \(L_{\text{aim}}\) and \(h_{0}\) is adjusted accordingly, limited by \(u_{h,\text{max}}\) for stability reasons.

The current load \(L\) is calculated as the force due to pressure on the deforming surface normal to the non-deforming surface. For a two-dimensional contact the load is specified in \(N/m\), for a three-dimensional contact the load is specified in \(N\).

To include a dependency of the timestep \(\Delta t\) a characteristic deformation time

\[
t_{cd} = \frac{R}{a_s}
\]  

(5.2)

is introduced, with \(R\) as the radius of the deforming body and \(a_s\) as the sonic velocity of the body, usually taken as \(a_s = 5000\ m/s\). The resulting change in gap reads

\[
\Delta h_{c0} = (w_{\text{max}} - w_{\text{min}}) \frac{L_{\text{aim}} - L}{L_{\text{aim}}} \frac{\Delta t}{t_{cd}} R_{def}
\]  

(5.3)

with \(w_{\text{max}}, w_{\text{min}}\) as the maximum and minimum Hertzian deflection in the domain and \(R_{def}\) as the under-relaxation factor for deflection. That has to be limited with the maximum rate of change of gap resulting in

\[
h_{c0} = h_{c0} + min\ [u_{h,\text{max}} \Delta t, |\Delta h_{c0}|] \ sign(\Delta h_{c0})
\]  

(5.4)

In this study it was found this approach worked quite well, usually keeping the load within 0.1% of the target load.
5.3 Isentropic EHL

In this section results are presented for the isentropic cavitation model of section 3.6 coupled together with the elastic deformation explained in chapter 4. For roller bearings typical operating conditions are chosen. In the first case a cylinder bearing is considered to analyse the outer flow field of an EHL contact. In the second case only half the domain is calculated as the main region of interest is the deforming EHL contact.

5.3.1 Full Cylinder

The case considered is a two-dimensional section of a cylinder bearing, the reduced Young’s modulus typical for a steel ball on steel ring contact. The sonic velocity of the vapour is set to \(a_v = 1000 \text{ m/s}\), which is clearly unphysical but necessary for stability as explained in section 3.6.3. The viscosity is a function of the isothermal Roelands viscosity-pressure relation and subject to shear-shinning as described by Johnson. The case parameters are listed in table 5.1.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder radius</td>
<td>(R = 10 \text{ mm})</td>
</tr>
<tr>
<td>Youngs modulus</td>
<td>(E_r = 3.45 \cdot 10^{11})</td>
</tr>
<tr>
<td>undeformed film thickness</td>
<td>(h_{c0} = -1.79 \mu m)</td>
</tr>
<tr>
<td>entrainment velocity</td>
<td>(u = 10 \text{ m/s})</td>
</tr>
<tr>
<td>dynamic viscosity liquid</td>
<td>(\mu_l = 10^{-2} \text{ m}^2/\text{s})</td>
</tr>
<tr>
<td>Roelands reference pressure</td>
<td>(p_{r,0} = 1.98 \cdot 10^{8} \text{ Pa})</td>
</tr>
<tr>
<td>Roelands pressure index</td>
<td>(z = 0.6)</td>
</tr>
<tr>
<td>Eyring stress</td>
<td>(\tau_0 = 7 \cdot 10^6 \text{ Pa})</td>
</tr>
<tr>
<td>dynamic viscosity vapour</td>
<td>(\mu_v = 8.97 \cdot 10^{-6} \text{ m}^2/\text{s})</td>
</tr>
<tr>
<td>environment pressure</td>
<td>(p_0 = 10^5 \text{ Pa})</td>
</tr>
<tr>
<td>vapour pressure</td>
<td>(p_{vapour} = 3000 \text{ Pa})</td>
</tr>
<tr>
<td>liquid density at (p_0)</td>
<td>(\rho_{l,sat} = 850 \frac{\text{kg}}{\text{m}^3})</td>
</tr>
<tr>
<td>vapour density at (p_{vapour})</td>
<td>(\rho_{v,sat} = 0.0288 \frac{\text{kg}}{\text{m}^3})</td>
</tr>
<tr>
<td>sonic velocity vapour</td>
<td>(a_v = 1000 \text{ m/s})</td>
</tr>
<tr>
<td>sonic velocity liquid</td>
<td>(a_l = 1200 \text{ m/s})</td>
</tr>
</tbody>
</table>

Table 5.1: Parameters - isentropic ehl - full cylinder

Figure 5.1 shows the computational domain, neglecting the curvature of the rings. The cylinder is accommodating the deflection on both the upper and lower contact. The Hertzian contact theory only applies to half-infinite bodies. Therefore the deflection calculation of the cylinder is split into an upper and a lower
half-cylinders. For each half-cylinder a reference point controlling the geometry is chosen, as discussed in chapter 4, and the undeformed film thickness for both half cylinders is $h_{c0} = -1.79 \, \mu m$, which would result in contacting surfaces if no fluid film were present. The movement of the reference points results in a slight stretching of the cylinder, which is negligible compared to the cylinder radius.

Figure 5.1: Computational domain - isentropic ehl - full cylinder

For density, the boundary conditions are zero gradient at all boundaries. Pressure boundary conditions are zero gradient for the walls and fixed at ambient pressure at the inlet and outlet. The velocity at the walls is set according to figure 5.1. At the inlet and outlet, velocity is calculated according to the mass-flux. All divergence terms are discretised using the upwind scheme. All gradient terms are evaluated using Gauss linear discretisation. The Laplacian terms are evaluated using Gauss linear corrected. The continuity equation (3.2) and temperature equation (3.87) are solved using the matrix solver BICCG (incomplete Cholesky preconditioned biconjugate gradient). The pressure equation (3.68) is solved using the ICCG (incomplete Cholesky preconditioned conjugate gradient) solver.

Figure 5.2 shows the velocity magnitude in the rolling case, where all surfaces are moving with $5 \, m/s$. The black lines with arrows are streamlines illustrating the circulation of fluid. In the lower part fluid is drawn into the contact region by the moving surfaces and then repulsed, since very little fluid transported through the contact. Figure 5.3 shows that effect in more detail, in the right figure one can see two streams of fluid at the walls going into the contact and the middle stream going out.
The resulting pressure and film thickness in the centre of the contact is shown in figure 5.4 with the typical constriction and the pressure spike.
Figure 5.4: Full cylinder - pressure and film thickness

Figure 5.5 shows vapour fraction in the cavitating region. The interface between vapour and liquid is not very sharp and the vapour region extends further downstream near the surfaces. Figure 5.6 shows the corresponding pressure in the cavitating region, reaching negative values of up to $-0.5 \, MPa$. Pressure is not varying significantly across the film thickness. The negative pressure predicted is in the realms of possibility as shown by Wissussek [52]. He measured negative pressures of up to $-0.78 \, MPa$ in a journal bearing.

In this study turbulent and surface tension effects are not considered and these two assumption shall be evaluated in this test case. The Reynolds number is a measure for the importance of inertial and viscous forces and is defined as

$$Re = \frac{\rho u l}{\eta}$$

where $u$ is the characteristic velocity, $l$ is the characteristic length, $\rho = 850 \, kg/m^3$ is the density and $\eta = 0.01 \, Pa \, s$ is the viscosity. The largest characteristic length scale in the centre of the contact is the extent of the cavitation bubble $l = 0.75 \, mm$ and the characteristic velocity is $u = 5 \, m/s$. Channel flow is laminar for $Re < 1350$, and fully turbulent for $Re > 18000$ according to Pope [65]. This leads to a Reynolds number of $Re_v = 319$ and thus laminar flow can be assumed in the cavitating region. In the centre of the contact characteristic length scale is smaller, so is the Reynolds number, and laminar flow can be assumed. The Reynolds number for the outer flow field with $l = 0.02 \, m$ is $Re_o = 8500$, which is inside the transitional
regime. This suggest that turbulent effects should be considered if one is interested in details of the outer flow field.

The Weber number is a measure for the importance of surface tension forces and inertial forces and is defined as

\[ We = \frac{\rho u^2 l}{\sigma_s} \]  

(5.6)

where \( u = 5 \text{ m/s} \) is the characteristic velocity, \( l \) is the characteristic length, \( \rho = 850 \text{ kg/m}^3 \) and \( \sigma_s = 0.02 \text{ N/m} \) is the surface tension. For the characteristic length the smallest possible value is the minimum film thickness and the largest value possible is the extent of the cavitation bubble. Then, \( l = [0.3 \mu m, 0.75 mm] \), which gives the range of the Weber number as \( We = [0.064, 160] \). According to Crowe [66] flow in minichannels with a diameter between 3 mm and 200 \( \mu m \) is inertia dominated for Weber numbers \( We \geq 11.0 \) and for lower numbers surface tension is important. This suggest that surface tension effects are important for the structure of the fluid in the cavitation region.
Figure 5.5: Full cylinder - cavitating region - vapour fraction

Figure 5.6: Full cylinder - cavitating region - pressure
5.3.2 Half Cylinder

Table 5.2 lists the case parameters and figure 5.7 shows the computational domain, which is half of a cylinder. This enables a saving of computing time based on the observation that the flow outside the contact does not influence the contact flow much. The Roelands and Eyring parameters are slightly different from the previous case and the elastic modulus is higher.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
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<tr>
<td>Youngs modulus</td>
<td>$E_r = 5.42 \cdot 10^{11}$</td>
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<tr>
<td>Load</td>
<td>$L = 3.45 \cdot 10^4 \text{N/m}$</td>
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<tr>
<td>entrainment velocity</td>
<td>$u = 1 \text{ m/s}$</td>
</tr>
<tr>
<td>dynamic viscosity liquid</td>
<td>$\mu_l = 10^{-2} \text{m}^2\text{s}^{-1}$</td>
</tr>
<tr>
<td>Roelands reference pressure</td>
<td>$p_{r,0} = 1.98 \cdot 10^{8} \text{Pa}$</td>
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<tr>
<td>Roelands pressure index</td>
<td>$z = 0.689$</td>
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<tr>
<td>Eyring stress</td>
<td>$\tau_0 = 5 \cdot 10^6 \text{Pa}$</td>
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<tr>
<td>dynamic viscosity vapour</td>
<td>$\mu_l = 8.97 \cdot 10^{-6} \text{m}^2\text{s}^{-1}$</td>
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<tr>
<td>environment pressure</td>
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<tr>
<td>vapour pressure</td>
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<tr>
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<td>$a_v = 1000 \text{ m/s}$</td>
</tr>
<tr>
<td>sonic velocity liquid</td>
<td>$a_l = 1200 \text{ m/s}$</td>
</tr>
</tbody>
</table>

Table 5.2: Parameters - isentropic ehl - half cylinder

Figure 5.7: Computational domain - isentropic ehl - half cylinder

For density, the boundary conditions are zero gradient at all boundaries. Pressure boundary conditions are zero gradient for the walls and fixed at ambient pressure at the inlet and outlet. The velocity at the walls is set according to figure 5.7. At the inlet and outlet, velocity is calculated according to the mass-flux. All divergence terms are discretised using the upwind scheme. All gradient terms
are evaluated using Gauss linear discretisation. The Laplacian terms are evaluated using Gauss linear corrected. The continuity equation (3.2) and temperature equation (3.87) are solved using the matrix solver BICCG (incomplete Cholesky preconditioned biconjugate gradient). The pressure equation (3.68) is solved using the ICCG (incomplete Cholesky preconditioned conjugate gradient) solver.

Figure 5.8 shows film-thickness and pressure distribution for pure rolling with both surfaces travelling at the same speed. Figure 5.9 shows the corresponding viscosity and shear-rate together with velocity vectors in the centre of the contact. The pressure is close to constant across the film thickness and is not displayed as a field plot. The velocity vectors in the very centre are uniform. At the exit constriction the fluid is accelerating which corresponds to the pressure gradient. As a result of this the local shear-rate is increasing at the top and bottom of the exit constriction. The viscosity distribution is largely driven by its dependence on pressure, increasing to about 7.50 Pas at the pressure spike from 0.01 Pas in ambient conditions. The local increase of shear-rate at the exit constriction does not have a major influence on viscosity.

Figure 5.10 shows film-thickness and pressure distribution for simple sliding with the cylinder travelling at 1m/s and the flat plane not moving. Figure 5.11 shows the corresponding viscosity and shear-rate together with velocity vectors in the centre of the contact. The pressure is close to constant across the film thickness and is not displayed as a field plot. The velocity distribution in the centre of the contact is close to linear and at the exit constriction the fluid is accelerating corresponding to the pressure gradient. The result of the velocity profile in the exit constriction is a comparably low shear-rate at the cylinder and and higher shear-rate at the flat plate. As a result the shear-rate is changing by one order of magnitude near the exit constriction across the film and it is non-uniform in the entire contact. The viscosity becomes non-uniform across the film because of this.

5.3.3 Closure

Results for a two-dimensional cylinder bearing have been presented, showing the outer flow field in conjunction with the deforming EHL contact. A dimensional analysis suggest that the fluid flow near the contact region and cavitating region is laminar. For detailed modelling of the outer flow field turbulence should be considered, which is not done in this study. The isentropic cavitation model predicts negative pressures in cavitating region which is in the realms of possibility. Fur-
thermore, a low Weber number suggest that surface tension effects are important for the structure of the fluid in the cavitation region.

Analysis of the flow field in the EHL contact shows that pressure is close to constant across the film thickness. Viscosity is close to constant across the film thickness in rolling, but varies across the film thickness in sliding caused by a complex distribution of the effect of shear-thinning.
Figure 5.8: Isentropic ehl - rolling

Figure 5.9: Isentropic ehl - rolling - viscosity and shear-rate
Figure 5.10: Isentropic ehl - sliding

Figure 5.11: Isentropic ehl - sliding - viscosity and shear-rate
5.4 Isothermal Isobaric EHL

5.4.1 Introduction

In the previous section an isentropic cavitation model was employed. In this section the isobaric cavitation model outlined in section 3.7 is coupled together with the elastic deflection of chapter 4. The reduced Young’s modulus is that of a steel on steel contact. A series of isothermal test cases are run to study the numerical stability of the developed algorithm and to explore a range of operating conditions. Four different case are attempted with two viscosities $\eta_1 = 1 Pa s$ and $\eta_2 = 0.01 Pa s$ and two entrainment velocities $u_{ent} = \frac{u_1 + u_2}{2}$, $u_{ent,1} = 2.5 m/s$ and $u_{ent,2} = 0.025 m/s$. The cases are listed in table 5.3.

The combination of $\eta_2 = 0.01 Pa s$ and $u_2 = 0.05 m/s$ could not be computed successfully, which will be explained later in this section.

For a line contact the following dimensionless parameters can be introduced [2].

$$G^* = \frac{E_r \alpha}{(5.7)}$$

$$W^* = \frac{L}{E_r R}$$

$$U^* = \frac{\eta_0 U_{avg}}{E_r R}$$

The materials group $G^*$ and the dimensionless load $W^*$ are constant in this case study. $U^*$ is the dimensionless parameter with the dominant influence on the film-thickness and is listed in table 5.3. $U^*$ is the same for case 1 and case 4, meaning with a piezo-viscous isothermal Reynolds solver the results would be the same, but in case 1 more shear-thinning occurs.

<table>
<thead>
<tr>
<th>Case</th>
<th>Viscosity [Pa s]</th>
<th>$u_{ent}[m/s]$</th>
<th>$U^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>2.5</td>
<td>$1.39 \cdot 10^{-11}$</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>0.025</td>
<td>$1.39 \cdot 10^{-13}$</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>2.5</td>
<td>$1.39 \cdot 10^{-9}$</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>0.025</td>
<td>$1.39 \cdot 10^{-11}$</td>
</tr>
</tbody>
</table>

Table 5.3: glass on steel - case overview

The common parameters for all cases are listed in table 5.4 with a reduced Young’s modulus of glass on steel and a diameter of the cylinder which is often used in experiments.
Table 5.4: glass on steel - case parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder radius</td>
<td>$R = 9.525, \text{mm}$</td>
</tr>
<tr>
<td>Young’s reduced modulus load</td>
<td>$E_r = 1.885 \times 10^{11} , \text{Pa}$</td>
</tr>
<tr>
<td>Roelands reference pressure</td>
<td>$L = 2 \times 10^5 , \text{N/m}$</td>
</tr>
<tr>
<td>pressure viscosity index</td>
<td>$p_{r,0} = 1.98 \times 10^8 , \text{Pa}$</td>
</tr>
<tr>
<td>thermoviscous constant</td>
<td>$\alpha_p = 1.88 \times 10^{-8} , \text{Pa,s}$</td>
</tr>
<tr>
<td>Eyring stress</td>
<td>$\beta = 0.0476/\text{K}$</td>
</tr>
<tr>
<td>dynamic viscosity vapour pressure</td>
<td>$\mu_l = 8.97 \times 10^{-6} , \text{m}^2/\text{s}$</td>
</tr>
<tr>
<td>environment pressure</td>
<td>$p_0 = 10^5 , \text{Pa}$</td>
</tr>
<tr>
<td>vapour pressure</td>
<td>$p_{\text{vapour}} = 5000 , \text{Pa}$</td>
</tr>
<tr>
<td>liquid density at $p_0$</td>
<td>$\rho_{l\text{,sat}} = 850 , \text{kg/m}^3$</td>
</tr>
<tr>
<td>vapour density at $p_{\text{vapour}}$</td>
<td>$\rho_{v\text{,sat}} = 0.0288 , \text{kg/m}^3$</td>
</tr>
<tr>
<td>Hertzian pressure</td>
<td>$p_{\text{hertz}} = 0.633 , \text{GPa}$</td>
</tr>
<tr>
<td>Hertzian contact radius</td>
<td>$a = 0.2 , \text{mm}$</td>
</tr>
</tbody>
</table>

For density, the boundary conditions are zero gradient at all boundaries. Pressure boundary conditions are zero gradient for the walls and fixed at ambient pressure at the inlet and outlet. The boundary conditions for velocity are set according to the slide-to-roll ratio. At the inlet and outlet, velocity is calculated according to the mass-flux. All divergence terms are discretised using the upwind scheme. All gradient terms are evaluated using Gauss linear discretisation. The Laplacian terms are evaluated using Gauss linear corrected. The continuity equation (3.2) and temperature equation (3.87) are solved using the matrix solver BICCG (incomplete Cholesky preconditioned biconjugate gradient). The pressure equation (3.68) is solved using the ICCG (incomplete Cholesky preconditioned conjugate gradient) solver.

5.4.2 Stability and Calculation Procedure

Generally, the thicker the film in the contact, the more stable is an ehl-calculation. A thicker film is more compressible and because changes in surface-geometry are smaller relative to the film-thickness, hence the effect on the fluid flow is less severe. Sliding conditions are more stable than rolling conditions. This is because the rate of shear-thinning of equation (2.36) is a function of the product of shear-rate and viscosity. In regions of high-pressure, the logarithmic pressure dependency is mitigated, since any jump in viscosity results immediately in more shear-thinning. For rolling conditions the rate of shear-thinning is generally smaller because of the
lower shear-rate. If the product of shear-rate and viscosity does not exceed the Eyring stress $\tau_0$, shear-thinning is essentially switched off.

**Start-up**

For each viscosity to be investigated, one case is taken to start calculations. Following the arguments above the most stable case for the lower viscosity is case 1 under sliding conditions. The most stable case for the higher viscosity is case 3 under sliding conditions. These two cases are started with an initial gap of $9.525 \cdot 10^{-7}m$ and then the two surfaces are pushed together at a maximum velocity of $u_{h,max} = 5 \cdot 10^{-4} m/s$. The timestep is $\Delta t = 5 \cdot 10^{-9}s$. The under-relaxation factor for the deflection is $R_{def} = 0.01$.

**Computational Time**

All computations are carried out on Dual Core AMD Opteron(tm) Processor 180 nodes. The cases are decomposed into two domains and run on one node resulting in a speed-up of 1.8. The two start-up cases took almost four days computational time each. The parallel performance is very much hardware and case dependent. In this case communication takes place between processors on the same motherboard, which is at the moment the fastest interconnect. If one uses nodes communicating across a network, the communication delay is higher and efficiency will drop. Larger cases usually scale better than smaller cases, as the computational time on each node in relation to the time of communication is higher. The way of decomposing a domain is important as well. One should minimise the number of boundary faces between processors in order to minimise the data to be communicated.

**Thin Film Failure**

As mentioned before case 2 could not be computed successfully. It was started from case 1 under isothermal sliding conditions and the velocity was reduced with a deceleration of $550m/s^2$. The timestep was subsequently reduced to $\Delta t = 10^{-9}s$ in combination with a deflection relaxation of $R_{def} = 0.005$, which is the most stable tried in a range of $R_{def} = [0.01 - 0.001]$. When a velocity of $u = 1.016m/s$ was reached a solution singularity occurred in the pressure equation, meaning that the matrix solver could not deliver a solution. Ten timesteps before that the maximum density started to oscillate from one timestep to another between a value in correlation with the equation-of-state and a bigger value than expected from the
pressure level. That oscillation grew until the singularity in the pressure equation occurred. The fluid/deflection solution saved shortly before that is smooth and seems to be valid. The reasons behind this instability could not be determined and requires further study.

**Stabilising Shear-thinning**

*Case 4* was started from its high-velocity counterpart *case 3* under isothermal sliding conditions, with the velocity being reduced with a deceleration of $550 \text{m/s}^2$. The timestep has to be reduced to $\Delta t = 2 \cdot 10^{-9} \text{s}$ when approaching the target velocity of $0.05 \text{m/s}$, otherwise pressure and deflection started to oscillate. The setup of *case 4* resulted in the same film-thickness as *case 1* having the same product of viscosity and velocity. For the latter a timestep of $\Delta t = 5 \cdot 10^{-9} \text{s}$ was stable and unlike *case 4* shear-thinning does take place.

The same effect, but more pronounced, was observed in the transition of *case 1* from isothermal sliding to isothermal rolling conditions. One surface was decelerated with $275 \text{m/s}^2$ as the other was accelerated with $275 \text{m/s}^2$, keeping the entrainment velocity constant and reducing the effect of shear-thinning. In order to keep the computational time in bounds and the solution stable, the timestep was subsequently reduced from $\Delta t = 5 \cdot 10^{-9} \text{s}$ down to $\Delta t = 5 \cdot 10^{-10} \text{s}$. The most stable deflection relaxation factor was $R_{\text{def}} = 0.01$, in this cases, with a tested range of $R_{\text{def}} = [0.02 - 0.001]$. This setup just about worked, there were still some fluctuations and the solution did not fully converge to a steady state. A good example for the pressure-deflection oscillation that occurred is given in figure 5.12, which is actually *stable*, meaning it continued to oscillate without crashing or producing a singularity in the fluid.

The transition from sliding to rolling of the other two cases did not pose a problem, as *case 3* had a much thicker film and *case 4* had no shear-thinning anyway.

**Deflection Relaxation**

In this case study and in other calculations not documented it was found that the deflection relaxation should be kept in a range of $R_{\text{def}} = [0.02 - 0.001]$. If $R_{\text{def}}$ was too high, the response to pressure variations was too quick, causing immediate oscillations. If it was too low, the pressure continued to rise and reaching unphysical levels, as the surface did not give way quickly enough, reinforced by the exponential
dependency of viscosity to pressure, which then resulted in too big response of the surface, resulting in slow pressure-deflection oscillations.

5.4.3 Case 1

Rolling

Pressure and film thickness for Case 1 with $\eta_0 = 0.01 \text{Pas}$ and $U = 5 \text{m/s}$ under rolling conditions are shown in figure 5.13. In figure 5.14 viscosity is shown together with shear-rate in the centre of the contact. The viscosity is close to uniform across the film and shear-thinning is essentially switched off. Pressure is close to constant across the film thickness.

Sliding

Pressure and film thickness for Case 1 with $\eta_0 = 0.01 \text{Pas}$ and $u_{ent} = 2.5 \text{m/s}$ under sliding conditions is shown in figure 5.15. In figure 5.16 viscosity is shown together with shear-rate in centre of the contact. The non-uniform distribution of viscosity across the film is due to shear-thinning as discussed in section 3.6. Pressure is close to constant across the film thickness. Figure 5.17 shows the cavitating region down-stream of the contact. The left figure showing the vapour fraction with a rather sharp interface at the end. The right figure shows the pressure distribution.
in the cavitating region, which is very close to the cavitation pressure. Figure 5.18 shows vapour fraction and velocity vectors at the end of the cavitation region. The extent of the cavitation zone coincides with the start of the fluid circulation in the exit region.
Figure 5.13: Case 1 - rolling - pressure and thickness

Figure 5.14: Case 1 - rolling - viscosity and shear-rate
Figure 5.15: Case 1 - sliding - pressure and thickness

Figure 5.16: Case 1 - sliding - viscosity and shear-rate
Figure 5.17: Case 1 - sliding - vapour fraction and pressure

Figure 5.18: Case 1 - sliding - cavitation end
5.4.4 Case 3

Rolling

Pressure and film thickness under isothermal rolling conditions is shown in figure 5.19. The distribution of viscosity and shear rate is shown in figure 5.20. There are small variations of viscosity across the film thickness in the centre of the contact and larger variations near the constriction, caused by shear-thinning. There are small variations of pressure across the film near the constriction, as shown in figure 5.21.

Sliding

Pressure and film thickness for Case 3 with $\eta_0 = 1 Pas$ and $U = 5 m/s$ under sliding conditions are shown in figure 5.23. The viscosity maximum (figure 5.24) is located more towards the inlet and upstream the pressure maximum where the shear-rate, hence shear-thinning, is locally smaller. Pressure is close to constant across the film thickness. The cavitation region shown in figure 5.24 extends much further to $x = 3 mm$ compared to Case 1 with $x = 0.7 mm$. 

105
Figure 5.19: Case 3 - rolling - pressure and thickness

Figure 5.20: Case 3 - rolling - viscosity and shear-rate
Figure 5.21: Case 3 - rolling - pressure and shear-rate

Figure 5.22: Case 3 - sliding - vapour fraction and velocity
Figure 5.23: Case 3 - sliding - pressure and thickness

Figure 5.24: Case 3 - sliding - viscosity and shear-rate
5.4.5 Case 4

Rolling

From a isothermal Reynolds theory point of view Case 4 with $\eta_0 = 1\text{Pas}$ and $U = 0.05\text{m/s}$ is identical to Case 1 with $\eta_0 = 0.01\text{Pas}$ and $U = 5\text{m/s}$ if viscosity is only a function of pressure and shear-thinning is not allowed. Hence, the calculations should show only minor differences in film thickness and pressure. In figure 5.25 pressure and film thickness for Case 4 under rolling conditions are compared to Case 1 and show only small differences. In figure 5.26 viscosity is shown together with shear-rate in the centre of the contact. The viscosity is close to uniform across the film and shear-thinning is essentially switched off. Pressure is close to constant across the film thickness.

![Figure 5.25: Case 4 - rolling - pressure and thickness](image)

Sliding

Pressure and film thickness for Case 4 under isothermal sliding conditions compared to Case 1 is shown in figure 5.27. The distribution of viscosity and shear-rate in the contact is shown in figure 5.28. The differences in film thickness and pressure are, again, small. In figure 5.28 viscosity is shown together with shear-rate in centre of the contact. The non-uniform distribution of viscosity across the film
Figure 5.26: Case 4 - rolling - viscosity and shear-rate

is due to shear-thinning as discussed in section 3.6. Pressure is close to constant across the film thickness.
Figure 5.27: Case 4 - sliding - pressure and thickness

Figure 5.28: Case 4 - sliding - viscosity and shear-rate
5.4.6 Summary

Stability

The range of stable operating conditions is limited by several factors. There is a high pressure limit in the range of 0.7 – 0.8 GPa maximum Hertzian pressure for the base viscosity $\eta_0 = 0.01 \, Pa\, s$. The mode of failure, if trying to go beyond that range, is a pressure-deflection oscillation as shown in figure 5.12. The same kind of oscillation has been reported in a Reynolds approach [34] and has been solved either by implicit pressure/film-thickness coupling [39] or by multigrid methods [30]. For the base viscosity $\eta_0 = 1 \, Pa\, s$ stability is generally better and the maximum achievable pressure was not further investigated.

There is also a limitation on the minimum film-thickness, which is far less investigated and only discussed in section 5.4.2. This will become important if one wants to move to boundary lubrication using a CFD approach, where fluid is entrained into a rough contact and the asperities of the two solids touch each other.

Results

Generally, in the rolling cases the viscosity was close to uniform across the film-thickness. In the sliding cases the viscosity varied across the film, which is caused by shear-thinning. In case of high velocity together with high viscosity, complex distributions of viscosity occurred with a local maximum occurring at the location of a local minimum in shear-rate. The pressure is generally close to constant across the film thickness, with the exception in case of thick film and pure rolling conditions, where small variations occurred.
5.5 Thermal Isobaric EHL

In this section results are presented for the isobaric cavitation model outlined in section 3.7 coupled together with the elastic deflection of chapter 4. The reduced Young’s modulus is that of a steel on steel contact. The developed algorithm is run for two viscosities $\eta_0 = 0.01 \, Pa \, s$ and $\eta_0 = 1 \, Pa \, s$, entrainment velocity $u_{ent} = \frac{u_1 + u_2}{2} = 2.5 \, m/s$ and three different slide-to-roll ratios $SRR = \frac{2|u_1 - u_2|}{u_1 + u_2} = [0, 1, 2]$. The cases with $\eta_0 = 0.01$ and $SRR = [0, 1, 2]$ are labelled A, B, C and are loaded with $L = 1 \cdot 10^5 \, N/m$. The cases with with $\eta_0 = 1$ and $SRR = [0, 1, 2]$ are labelled D, E, F and are loaded with $L = 2 \cdot 10^5 \, N/m$. The isothermal cases are compared to the Reynolds theory. The effect of temperature is then considered by comparison to the isothermal results. The common parameters for all cases are listed in table 5.5. The thermal properties are listed in table 5.6.

The heat generated in an ehl contact can be very significant and the temperature distribution of the fluid is strongly coupled with the temperature of the bounding solids. In section 2.2.5 the Carslaw – Jaeger boundary condition for moving surfaces and a boundary condition for stationary surfaces has been introduced. The thermal properties of the bounding solids are listed in table 5.7. For the Carslaw – Jaeger condition the Peclet number needs to be evaluated to verify its applicability. For the cases with $SRR = 1$ the slowest surface velocity is $U_s = 1.25 \, m/s$. This results in a Peclet number of $Pe = \frac{a U_s}{\alpha T} = 19$. This is above the limit of applicability of $Pe > 5$ set by Johnson [3].

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder radius</td>
<td>$R = 10$</td>
<td>mm</td>
</tr>
<tr>
<td>Young’s reduced modulus</td>
<td>$E_r = 3.4523 \cdot 10^{11}$</td>
<td>Pa</td>
</tr>
<tr>
<td>Roelands reference pressure</td>
<td>$p_{r,0} = 1.98 \cdot 10^8$</td>
<td>Pa</td>
</tr>
<tr>
<td>pressure viscosity index</td>
<td>$Z = 0.689$</td>
<td></td>
</tr>
<tr>
<td>thermoviscous constant</td>
<td>$\beta = 0.0476$</td>
<td>1/K</td>
</tr>
<tr>
<td>Eyring stress</td>
<td>$\tau_0 = 5 \cdot 10^6$</td>
<td>Pa</td>
</tr>
<tr>
<td>dynamic viscosity vapour</td>
<td>$\mu_l = 8.97^{-6}$</td>
<td>$\frac{m^2}{s}$</td>
</tr>
<tr>
<td>environment pressure</td>
<td>$p_0 = 10^5$</td>
<td>Pa</td>
</tr>
<tr>
<td>vapour pressure</td>
<td>$p_{vapour} = 5000$</td>
<td>Pa</td>
</tr>
<tr>
<td>liquid density at $p_{vapour}$</td>
<td>$\rho_{l, sat} = 850$</td>
<td>$\frac{kg}{m^3}$</td>
</tr>
<tr>
<td>vapour density at $p_{vapour}$</td>
<td>$\rho_{v, sat} = 0.0288$</td>
<td>$\frac{kg}{m^3}$</td>
</tr>
</tbody>
</table>

Table 5.5: Case parameters

The computational domain is shown in figure 5.29. For density, the boundary conditions are zero gradient at all boundaries. Pressure boundary conditions are
<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>heat capacity liquid</td>
<td>$C_{p,l} = 2300$</td>
<td>J/(kg K)</td>
</tr>
<tr>
<td>heat capacity vapour</td>
<td>$C_{v,l} = 1800$</td>
<td>J/(kg K)</td>
</tr>
<tr>
<td>ambient temperature</td>
<td>$T_0 = 353$</td>
<td>K</td>
</tr>
<tr>
<td>thermal conductivity liquid</td>
<td>$k_l = 0.15$</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>thermal conductivity vapour</td>
<td>$k_v = 0.025$</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>heat of evaporation</td>
<td>$h_{evap} = 287$</td>
<td>kJ/kg</td>
</tr>
</tbody>
</table>

Table 5.6: Thermodynamic parameters - fluid

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>density solid</td>
<td>$\rho_s = 7850$</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>heat capacity solid</td>
<td>$C_{v,s} = 450$</td>
<td>J/(kg K)</td>
</tr>
<tr>
<td>thermal conductivity solid</td>
<td>$k_s = 47$</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>thermal diffusivity solid</td>
<td>$\alpha_{T,s} = \frac{k_s}{\rho_s C_s} = 1.31 \cdot 10^{-5}$</td>
<td>W/(mK)</td>
</tr>
</tbody>
</table>

Table 5.7: Thermodynamic properties - solid

zero gradient for the walls and fixed at ambient pressure at the inlet and outlet. The boundary conditions for velocity are set according to the slide-to-roll ratio. At the inlet and outlet, velocity is calculated according to the mass-flux. All divergence terms are discretised using the upwind scheme. All gradient terms are evaluated using Gauss linear discretisation. The Laplacian terms are evaluated using Gauss linear corrected. The continuity equation (3.2) and temperature equation (3.87) are solved using the matrix solver BICCG (incomplete Cholesky preconditioned biconjugate gradient). The pressure equation (3.68) is solved using the ICCG (incomplete Cholesky preconditioned conjugate gradient) solver.

![Figure 5.29: Computational domain](image)

All computations were carried out on Dual Core AMD Opteron(tm) Processor 180 nodes. The cases consisting of 4370 hexahedrons with ten cells across the film are decomposed into two domains and run on one node with a speed-up of 1.8
compared the single processor time. To start up an isothermal case with $SRR = 2$ and bring it to the target load took almost four days. The cases with $SRR = [0, 1]$ were started from the simple sliding cases and the surface velocities were gradually changed, which took a day. The thermal calculations were started from their isothermal counterpart and they also took about a day to converge. Depending on the case the timestep was chosen in a range of $\Delta T = [5 \cdot 10^{-10} s, 10^{-8} s]$, which is the reason for the long calculation times. Currently, the maximum Hertzian pressure achievable with the developed method is around 0.8 GPa. The deflection calculation is under-relaxed with $R_{def} = 0.01$. For the cases with the highest viscous heating it was necessary to under-relax the temperature with a factor of $R_T = 0.01$.

### 5.5.1 Isothermal Results

Figure 5.30 show isothermal film thickness and pressure results in comparison to the Reynolds results. The Reynolds solution is supplied by Dumont [67] using the approach outlined by Venner [30] employing the viscosity models described by Johnson [11] and Conry [10]. The low-viscosity cases A, B and C show typical EHL behaviour with a constriction at the rear of the contact. Case A has a small pressure spike just before the end of constriction which disappears in sliding conditions (case B and C). The film-thickness is almost unaffected by the slide-to-roll ratio.

The high-viscosity cases D, E and F show the behaviour for thick-film ehl-contacts. In pure rolling conditions (case D) there is a large pressure spike and a constriction at the end of the contact. In sliding conditions (case E and F) the pressure spike and constriction disappear. The maximum pressure drops from 1.25 GPa to around 0.5 GPa. The minimum film thickness falls with rising slide-to-roll ratio.

There is very good agreement between the Reynolds and the CFD solution for all cases except case D, where the pressure spike and the onset of the constriction of the CFD approach is shifted slightly downstream. Possible reasons for that are outlined in the discussion.

Details of the isothermal CFD solutions are shown in figures 5.31, 5.32 and 5.33. For each case the dynamic viscosity $[Pa s]$ is shown in the upper part of the picture and the shear-rate $[s^{-1}]$ together with velocity vectors is shown in the lower part of the picture.

For case A, thin-film pure rolling, the viscosity is constant across the film with
a maximum viscosity of $27 \text{Pa s}$. High shear-rates only occur in the constriction, where the fluid has to accelerate, and are low otherwise, so that essentially little shear-thinning takes place. Pressure does not vary significantly across the film-thickness.

For case $B$, $SRR = 1$, there is a considerable level of shear-thinning in the centre of the contact resulting in the reduction of the maximum viscosity by more than an order of magnitude to $1.91 \text{Pa s}$. The fluid is accelerating at the constriction, which results in maximum shear-rate occurring at the upper, slower surface. Towards the lower, faster surface there is a local minimum as the velocity gradient induced by the Couette flow is cancelled out by the fluid acceleration at the constriction. The complex distribution of shear-rate in the contact causes the viscosity to vary across the film, which can be seen by the tilting of isolines of viscosity. Pressure does not vary significantly across the film-thickness.

For case $C$, simple sliding with the upper surface stationary, there is a further reduction in the maximum viscosity to $1.09 \text{Pa s}$ caused by more shear-thinning. Although the changes in viscosity are large between sliding and rolling conditions, the effect on film-thickness and pressure is small, as shear-thinning in the centre of the contact has little effect on the amount of fluid entrained upstream. The maximum shear-rate occurs at the upper, slower surface. The local minimum of shear-rate moves to the surface of the lower, faster surface. This is again caused by the mix of Couette flow and fluid acceleration at the constriction. The complex distribution of shear-rate in the contact again causes the viscosity to vary across the film. Pressure does not vary significantly across the film-thickness.

For case $D$, thick film pure rolling and twice the load of the thin film case, the viscosity varies slightly across the film in the centre of the contact and varies even more just before the constriction. This is caused by shear-thinning. The maximum viscosity of $97500 \text{Pa s}$ at the location of the pressure spike is at the lower, non-deforming surface, whereas at the upper surface the viscosity is about $35000 \text{Pa}$. There are small pressure variations across the film thickness which will be discussed in section 5.5.2.

Case $E$, $SRR = 1$, results in a complex flow pattern where the distribution of viscosity and shear-rate is highly varying across the film. The maximum viscosity is reduced by two orders of magnitude to $1089 \text{Pa s}$, and is not at the location of the maximum pressure, but upstream, where the effect of shear-thinning is locally smaller. Another local maximum of viscosity is at the lower surface coinciding roughly with the location of the pressure spike. The resulting velocity gradients
across the film are only close to linear near the origin and show different behaviour otherwise. There are very small pressure variations across the film thickness.

For Case F, simple sliding, the maximum viscosity is further reduced to $202 \text{Pa s}$ and its location moved further upstream compared to case E. The pressure spike has disappeared and there is only a weak second local maximum near the origin. There are very small pressure variations across the film thickness.

Figure 5.30: Isothermal - pressure and film thickness (Note that the x-axis is larger in D, E, F than in A, B, C and that the pressure scale in D differs from E and F)
Figure 5.31: Isothermal - case A and D - dynamic viscosity, shear-rate and velocity
Figure 5.32: Isothermal - case B and E - dynamic viscosity, shear-rate and velocity
Figure 5.33: Isothermal - case C and F - dynamic viscosity, shear-rate and velocity
5.5.2 Thermal Cases

Pressure, measured at the upper deforming surface, and film thickness of the thermal cases are compared to their isothermal counterparts in figure 5.34. Details of the thermal flow fields are shown in figures 5.35 and 5.36. For each case the temperature \([K]\) is shown in the upper part, the viscosity \([Pa\ s]\) and pressure \([Pa]\) are shown in the middle part and the shear-rate \([s^{-1}]\) together with velocity vectors are shown in the lower part of the picture.

For case \textit{A}, thin film and pure rolling, the thermal effects are small, with a maximum temperature rise of 0.41 \(K\) at the constriction. There is a small reduction of maximum viscosity of 1.26\(Pas\) compared to the isothermal case. The pressure distribution and the film thickness are hardly changed. Viscosity and pressure do not vary significantly across the film-thickness.

For case \textit{B}, \(SRR = 1\), the temperature rises in the centre of the contact by up to 15 \(K\). The location of maximum temperature is slightly towards the surface of the cylinder, which is the slower and thus hotter surface. The localised high temperature between the surfaces results in a lower viscosity which leads to the development of a shear band in the middle of the contact. These higher shear-rates result in more shear-thinning, causing a further drop of viscosity. The maximum viscosity of 2.61\(Pas\) occurs on the rigid wall, the slower surface. Thus it is higher than the maximum viscosity of 1.91\(Pas\) in the isothermal case, because, with the shear band present the shear-rates and shear-thinning at the surfaces are lower. There is a small drop in film thickness compared to the isothermal case and a small change in pressure distribution just before the constriction. Pressure does not vary significantly across the film-thickness.

For case \textit{C}, thin film and simple sliding, the temperature rises in the centre of the contact by up to 26 \(K\). The location of maximum temperature is at the upper, stationary surface. This temperature rise results in lower viscosities at the upper wall, leading to a shear band attached to it. The maximum viscosity of 1.79\(Pas\) occurs on the lower wall, which is higher than the 1.09\(Pas\) value in the isothermal case, again caused by lower shear-rates. The heat-flux in the stationary surface is conducted upstream and downstream of the contact, which leads to a lower viscosity level in the inlet region, which results into less lubricant being dragged into the contact, leading in turn to a significant drop in film-thickness. Pressure does not vary significantly across the film thickness.

For case \textit{D}, thick film and pure rolling, the temperature rises by up to 29 \(K\)
at the constriction. In the centre of the contact, between the two hot spots, the generated heat is largely conducted into the solids. A considerable fraction of the heat generated at the constriction is convected out into the region downstream. The maximum viscosity of $1.2 \cdot 10^5 \text{Pas}$ occurs at the location of the pressure spike at the lower surface, and is around a quarter higher than for the isothermal case. The viscosity distribution in the inlet region is significantly lower, resulting in generally lower film thickness than in the isothermal case. The pressure spike is shifted slightly upstream and is about $0.25 \text{GPa}$ larger and there are small variations of pressure across the film-thickness near its origin.

For case $E$, $SRR = 1$, the temperature rises in the centre of the contact by up to $85 \text{K}$. The location of maximum temperature is towards the surface of the cylinder, which is the slower and thus hotter surface. The high temperature in the middle between the surfaces results in a lower viscosity which leads to the development of a marked shear band in the middle of the contact. These higher shear-rates result in more shear-thinning causing a further drop of local viscosity. The maximum viscosity of $1.1 \cdot 10^8 \text{Pas}$ occurs on the rigid wall, which is the faster surface. This is five orders of magnitude higher than in the isothermal case, because, with the shear band present, the shear-rates and thus shear-thinning at the surfaces is much lower and the pressure level is higher. The pressure distribution has changed completely compared to the isothermal case. The maximum pressure rises from $0.55 \text{GPa}$ to $0.9 \text{GPa}$ with a pressure plateau at the centre of the contact and much steeper pressure gradients. Pressure varies significantly, in contrast to the isothermal case, across the film thickness in the centre of the contact. The pressure level is lower near than surfaces than in the middle between the surfaces which is a result of the very large viscosity gradients and shear-rates in that region. The film thickness is reduced significantly as less fluid is entrained in the inlet region due to lower viscosity.

For case $F$, simple sliding, the temperature rises in the centre of the contact by up to $91 \text{K}$. The location of maximum temperature is close to the upper, stationary surface. This temperature rise results in lower viscosities at the upper wall leading to a shear band close to it. The maximum viscosity of $1.6 \cdot 10^7 \text{Pas}$ occurs on the lower wall, which is five orders of magnitude higher than in the isothermal case, again caused by lower shear-rates and higher pressure level. The heat-flux into the stationary surface is conducted upstream and downstream of the contact, which leads to a lower viscosity level in the inlet region, which results into less lubricant being drag into the contact leading to a significant drop in film-thickness. The film
thickness is about half that of the isothermal case. The pressure distribution has changed completely compared to the isothermal case. The maximum pressure rises from 0.5\(\text{GPa}\) to 0.8\(\text{GPa}\) with much steeper pressure gradients. Pressure varies significantly, in contrast to the isothermal case, across the film thickness in the centre of the contact. The pressure level is lower near the lower surfaces than in the middle and upper part which is a result of the large viscosity gradients near the lower surface.

Figure 5.34: Thermal - pressure and film thickness  (Note that the x-axis and pressures scales differ in D, E, F from A, B, C)
Figure 5.35: Thermal - case A - temperature, dynamic viscosity, shear-rate and velocity
Figure 5.36: Thermal - case B - temperature, dynamic viscosity, shear-rate and velocity
Figure 5.37: Thermal - case $C$ - temperature, dynamic viscosity, shear-rate and velocity
Figure 5.38: Thermal - case $D$ - temperature, dynamic viscosity, shear-rate and velocity
Figure 5.39: Thermal - case E - temperature, dynamic viscosity, shear-rate and velocity
Figure 5.40: Thermal - case $F$ - temperature, dynamic viscosity, shear-rate and velocity.
Friction

One important aim of EHL modelling is to predict friction forces accurately. The friction coefficient is defined as

\[ \mu_F = \frac{F_f}{L} \] (5.10)

where \( L \) is the load and \( F_f \) is the friction force. Table 5.8 summarises the resulting friction coefficients at the rigid wall for all cases. In the case of pure rolling the friction force increases slightly for the thermal cases compared to the isothermal ones by 11.1\% and 13.2\% for cases A and D. For the low-viscosity cases B and C the friction reduces by 45.1\% and 69.2\% respectively, which results in a lower friction force for case C than for case B. This is caused by the higher temperature level and thus lower viscosity. For the high-viscosity cases E and F the friction reduces by 84.1\% and 88.5\%, which results again in a lower friction force for the case with more sliding (case F). This is caused again by the higher temperature level and thus lower viscosity. The distribution of wall shear-stresses at the rigid wall is shown in figure 5.41 for all cases. For case A the difference are not visible on this scale. For case B and C the shape are similar but the level of the shear-stresses are different. For case D, high viscosity and pure rolling, the shape is similar but the amplitude increased in the thermal case and the peaks are shifted upstream. For the high viscosity and sliding cases E and F the thermal stress curves show no obvious relation to their isothermal counterparts as their flow-fields and resulting friction forces are very different.

<table>
<thead>
<tr>
<th>case</th>
<th>( \eta_0 )</th>
<th>SRR</th>
<th>( \mu_F ) isoT</th>
<th>( \mu_F ) T</th>
<th>change [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.01</td>
<td>0</td>
<td>0.0004</td>
<td>0.0005</td>
<td>11.1</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>1</td>
<td>0.0371</td>
<td>0.0203</td>
<td>-45.3</td>
</tr>
<tr>
<td>C</td>
<td>0.01</td>
<td>2</td>
<td>0.0220</td>
<td>0.0068</td>
<td>-69.2</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0</td>
<td>0.0072</td>
<td>0.0082</td>
<td>13.3</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>1</td>
<td>0.1106</td>
<td>0.0176</td>
<td>-84.1</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>2</td>
<td>0.1268</td>
<td>0.0145</td>
<td>-88.5</td>
</tr>
</tbody>
</table>

Table 5.8: friction force

Cavitation

The cavitation model employed is quite similar to the Reynolds exit condition where the pressure is not allowed to drop below zero. The pressure inside the cavitation region is maintained around the specified cavitation pressure \( p_{cav} = 5000 \text{ Pa} \). Figure 5.42 shows the vapour fraction and pressure around the cavitating
Figure 5.41: wall shear stresses at rigid wall

zone for the isothermal case $C$, $SRR = 2$. Cavitation starts after the constriction and ends at about $x = [0.7, 0.8] \text{mm}$. The interface between vapour and liquid is distinct but not very sharp. The pressure inside the cavitation region is very close to the cavitation pressure $p_{\text{cav}}$. Figure 5.43 shows the vapour fraction and pressure around the cavitating zone for the isothermal case $D$, $SRR = 0$. The general features are essentially the same as in case $C$, but the cavitation region extents to $x = [3, 3.7] \text{mm}$.

Figure 5.42: Cavitation zone case $C$    Figure 5.43: Cavitation zone case $D$

5.5.3 Discussion

Summary of Isothermal Results

In the sliding cases the viscosity varies significantly across the film in the centre of the contact due to shear-thinning. The isothermal high viscosity results show complex flow patterns with very high gradients of viscosity and shear-rate across
the fluid film. The isothermal, low viscosity cases show no significant pressure variation across the film-thickness and agree very well with the Reynolds theory. The isothermal, high viscosity cases show small pressure variations across the film thickness. In case of sliding there is very good agreement with the Reynolds approach. The pressure variations across the film thickness in the rolling cases are more significant than when sliding is present. The rolling case displays small differences from the Reynolds theory.

**Summary of Thermal Results**

The thermal, low viscosity results show that, in sliding conditions a temperature-induced shear band develops which is closer towards the slower moving and thus hotter surface. In rolling conditions the temperature increase is not very significant.

The thermal, high viscosity results show significant heating, even in pure rolling conditions. Friction force predictions are very much affected by thermal effects, with differences up to −88.5% compared to the isothermal case. In the case of sliding, a pronounced temperature-induced shear band evolves closer towards the hotter surface. Compared to the isothermal case, the shape of the pressure distributions are very different and have steeper gradients.

The thermal, high viscosity cases show no significant pressure variation across the film-thickness. The thermal, high viscosity cases show small pressure variations across the film thickness in the case of rolling and large pressure variations in the case of sliding.

**Analysis**

Of particular interest from the CFD results is that there are significant variations in pressure across the film thickness for the high viscosity cases. This is, of course, contrary to Reynolds theory. The driving force behind these pressure variations can be investigated by considering the momentum equation (3.3) which is simplified by neglecting the time derivative and the convection term as the calculations are steady-state and convection was found not to be significant. Expanding the remaining terms leads to

\[
\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left( \eta \frac{\partial u}{\partial y} \right)_{x_A} + \frac{\partial}{\partial y} \left( \eta \frac{\partial v}{\partial x} \right)_{x_B} + \frac{\partial}{\partial x} \left( 2\eta \frac{\partial u}{\partial x} \right)_{x_C} - \frac{\partial}{\partial x} \left( \frac{2}{3} \eta \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \right)_{x_D} \tag{5.11}
\]
\[ \frac{\partial p}{\partial y} = \frac{\partial}{\partial y} \left( 2\eta \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial x} \left( \eta \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial v}{\partial x} \right) - \frac{\partial}{\partial y} \left( \frac{2}{3} \eta \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \right) \] (5.12)

where \( x \) is the spatial coordinate along the flow direction, \( y \) is the spatial coordinate across the film thickness, \( u \) is the velocity component along \( x \) and \( v \) is the velocity component along \( y \). Equation (5.11) is the basis of the Reynolds equation, but only the term \( X_A \) is generally considered on the right hand side. Equation (5.12) is always neglected in the Reynolds approach. The CFD approach shows, however, that these assumptions are invalid in some conditions where a variation of pressure across the film thickness occurs.

Figure 5.44 compares the absolute pressure derivatives with the absolute values of the viscous stresses for both the isothermal case \( D \) (which showed small pressure variations across the film thickness and differences to the Reynolds approach) and the thermal case \( F \) (which showed large pressure variations across the film thickness). The same scale used for the pressure derivative is used for the respective viscous stresses. The terms \( X_C \) and \( Y_C \) have the smallest contribution compared to the other terms and are not displayed.

The isothermal case \( D \), pure rolling, shows small pressure variations across the film thickness and their origin can be seen by considering the variations of the terms in figure 5.44. In the flow direction (equation (5.11)) the term \( X_A \) is dominant, which is in agreement with the Reynolds theory. However, the terms \( X_B \) and \( X_D \) also have significant contributions in the centre of the contact, especially at the onset of the constriction. In the direction across the film-thickness (equation (5.12)) the term \( Y_A \) is dominant. Term \( Y_D \) is very significant and counteracts term \( Y_A \) as it contains a third of term \( Y_A \) and has opposite sign. Term \( Y_B \) contributes significantly near the onset of the constriction. Overall, \( X_A \) and \( Y_A \) have a predominant influence, but the less important terms contribute at the onset of the constriction. Since this may be interpreted as a surface feature, this suggests that these terms are likely to grow in importance in rough contacts.

The thermal case \( F \), simple sliding, shows large pressure variations across the film thickness and their origin can be seen by considering the variations of the terms in the right hand side of figure 5.44. In the flow direction (equation (5.11)) the term \( X_A \) is dominant, which is in agreement with the Reynolds theory. The term \( X_B \) is locally important, near the lower surface in the region of high pressure. Term \( X_D \)
is not very significant. In the direction across the film-thickness (equation (5.12)) the term $Y_A$ is dominant. Term $Y_D$ is very significant and again counteracts term $Y_A$ as it contains a third of term $Y_A$ and has opposite sign.

The reason why the pressure variation across the film, $\frac{\partial p}{\partial y}$, is much greater in the thermal case than the isothermal case is because of the variation of viscosity across the film, which results in large part from the temperature gradient across the film. This leads to a stronger variation of all viscous terms across the film, in particular term $Y_A$.

### 5.5.4 Conclusion

Isothermal and thermal test cases under moderate loads and steady-state conditions were presented. Very good agreement was found with the Reynolds theory in most cases. For high viscosity and high velocity rolling conditions there were small differences to the Reynolds theory due to viscous stress terms not considered in the Reynolds approach. All thermal calculations under sliding conditions developed a temperature-induced shear-band which is closer towards the slower, and thus hotter, surface. The thermal, high viscosity calculations under sliding conditions showed significant pressure variation across the film thickness due to very large viscosity gradients induced by temperature gradients across the film thickness. The impact of temperature on the friction force is very significant.
Figure 5.44: Viscous stresses - isothermal case D and thermal case F
5.6 3D EHL

In this section a three-dimensional point contact is modelled, coupling the isobaric cavitation model together with the deflection algorithm described in section 4.4.

Parts of the computational mesh is shown in figure 5.45 which consists of a quarter-sphere with $R = 10 \text{ mm}$ on a moving wall, taking advantage of the symmetry-plane along the main flow-direction. All other faces, which are not shown in figure 5.45, are grouped together as the open boundary. The quarter-sphere is accommodating the deflection, the moving wall is rigid. The computational domain is 120 mm long in the flow direction and 15 mm wide perpendicular to the symmetry plane.

For density, the boundary conditions are zero gradient at all boundaries. Pressure boundary conditions are zero gradient for the walls and fixed at ambient pressure at the open boundary. The velocity at the moving is set according to figure 5.45 with $u = 1 \text{ m/s}$. At the open boundary velocity is calculated according to the mass-flux. All divergence terms are discretised using the upwind scheme. All gradient terms are evaluated using Gauss linear discretisation. The Laplacian terms are evaluated using Gauss linear corrected. The continuity equation (3.2) and temperature equation (3.87) are solved using the matrix solver BICCG (incomplete Cholesky preconditioned biconjugate gradient). The pressure equation (3.68) is solved using the ICCG (incomplete Cholesky preconditioned conjugate gradient) solver.

The case parameters are listed in table 5.9 with a reduced Young’s modulus of glass on steel. Temperature effects are not considered. The computational mesh used is the one described in section 4.4 with about 200,000 cells.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder radius</td>
<td>$R = 10 \text{ mm}$</td>
</tr>
<tr>
<td>Young’s reduced modulus load</td>
<td>$E_r = 1.9 \cdot 10^{11} \text{ Pa}$</td>
</tr>
<tr>
<td></td>
<td>$L = 3.19 \text{ N/m}$</td>
</tr>
<tr>
<td>Roelands reference pressure</td>
<td>$p_{r,0} = 1.98 \cdot 10^8 \text{ Pa}$</td>
</tr>
<tr>
<td>pressure viscosity index</td>
<td>$\alpha = 1.88 \cdot 10^{-8} \text{ Pa s}$</td>
</tr>
<tr>
<td>Eyring stress</td>
<td>$\tau_0 = 5 \cdot 10^6 \text{ Pa}$</td>
</tr>
<tr>
<td>dynamic viscosity vapour</td>
<td>$\mu_l = 8.97^{-6} \text{ m}^2/\text{s}$</td>
</tr>
<tr>
<td>environment pressure</td>
<td>$p_0 = 10^5 \text{ Pa}$</td>
</tr>
<tr>
<td>vapour pressure</td>
<td>$p_{vapour} = 5000 \text{ Pa}$</td>
</tr>
<tr>
<td>liquid density at $p_0$</td>
<td>$\rho_{l,sat} = 850 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>vapour density at $p_{vapour}$</td>
<td>$\rho_{v,sat} = 0.0288 \text{ kg/m}^3$</td>
</tr>
</tbody>
</table>

Table 5.9: 3d - case parameters
5.6.1 Calculation Procedure

The start-up procedure is the same as for the case study in section 5.4. Starting with an initial gap of $10^{-7}m$ the two surfaces are pushed together, limited with a maximum velocity of $2 \cdot 10^{-4}m/s$. The initial timestep was $10^{-8}s$, which was subsequently reduced to $10^{-9}s$. The deflection relaxation factor was initially 0.01, which was subsequently reduced to 0.001. The calculation time took approximately 3 weeks on 32 processors (Intel Xeon 5100).

5.6.2 Results

Figure 5.46 shows the resulting surface geometry, with a minimum film-thickness of $5 \cdot 10^{-8}m$ and an increment of $5 \cdot 10^{-9}m$. The flow is entrained from the left forming the typical horseshoe shape for the film-distribution with a plateau at the centre. The resulting pressure distribution is shown in figure 5.47 with a maximum pressure of 0.25GPa and a increment of 25MPa. The pressure shows some oscillation in the centre of the contact, which are most likely due to numerical instabilities as the solution would not settle down to a steady state.

The cavitation region starting downstream of the constriction is shown in figure 5.48. It forms a bubble which is 2-3 times the size of the contact radius. There is a drop in vapour fraction at the end of cavitation region, which is, again, most likely due to numerical instabilities.

Figure 5.49 shows the pressure distribution and surface deformation, the horseshoe shape is clearly visible, in the centre of the contact. Velocity vector together with the vapour fraction are shown in the symmetry plane showing the start of the cavitation region just downstream the constriction.

Figure 5.50 displays the shear-rate at the deforming body and two isosurfaces of shear-rate, showing a complex three-dimensional shape in the centre of the contact.

Figure 5.51 shows the extent of the cavitation bubble and two sets of streamlines. The yellow set is going through the contact where the stream-lines are slightly diverted out of the contact. The white set of stream-lines is placed further away from the rigid wall and is started upstream the contact region. They are all diverted around the contact region, as only very little fluid can go through.
5.6.3 Closure

The results presented should be seen as a proof-of-concept for realising three-dimensional calculations of an EHL contact. Much more work is necessary on the grid to improve speed and accuracy. The fluid and deflection equations are solved throughout the entire domain every time-step, which may not be necessary for the region far from the contact. Ways of improving the 3D solution are discussed in chapter 6 in more detail.
Figure 5.46: 3D - film thickness

Figure 5.47: 3D - pressure
Figure 5.48: 3D - vapour fraction

Figure 5.49: 3D - pressure and vapour fraction
Figure 5.50: 3D - shear-rate and vapour fraction

Figure 5.51: 3D - streamlines and cavitation bubble
Chapter 6

Future Development

6.1 Transient Calculations

So far all calculations presented were steady-state simulations with smooth surfaces. In reality the conditions in most ehl-contacts are highly time-dependent. The applied loads and the surface velocities can change. Real surfaces are usually rough, which can have a big impact on the transient behaviour.

In transient calculations the fluid boundary is deforming in time, which the computational cells have to accommodate. For a moving mesh the discretisation of the governing equations needs to be adapted with $U_b$ as the velocity of the control volumes. The convection term in equation 3.10 becomes

$$\int_V \nabla \cdot (\rho (U - U_b) \phi) dV = \sum_f S_f \cdot (\rho (U - U_b)_f) \phi_f$$

(6.1)

If $U_b$ is only evaluated based on the movement of the computational mesh, mass conservation can be violated. In order to enforce mass conservation, $U_b$ has to fulfil the space conservation law, which states that the change in volume of a cell has to equal the volume swept by the faces of a cell and is explained in detail by Ferziger [40].

$$\frac{d}{dt} \int_V dV - \int_S U_b \cdot n \, dS = 0$$

(6.2)

Another important detail is the treatment of the time derivate when dealing with a compressible fluid. When deriving the pressure equation for the isentropic cavitation model in section 3.7.1 the constant term $\rho_{l,0}$ in equation 3.51 is dropped, which is mathematically correct. However, if mesh movement is present the control
volume $V$ changes in time. The time derivative is integrated over the changing control volume $V$ following equation [3.14] and the term $\rho_l,0$ can no longer be dropped. The pressure equations derived in chapter 3 have to be modified accordingly for transient calculations.

The elastic deformation algorithm of chapter 4 is based on the Hertzian contact theory assuming a steady state for both the solid and the pressure distribution on the deforming body. As the deflection is under-relaxed for stability reasons, the forward iterative algorithm outlined in chapter 5 should modified, so that fluid and solid solution are iterated within a time step until an equilibrium is reached.

Glovnea et al. carried out experiments on a single transverse ridge in a circular ehl-contact [68]. They found in pure rolling that the ridge recovered to about 90 percent of its undeformed height. When the slide/roll ratio $SRR = \frac{2|u_1 - u_2|}{u_1 + u_2}$ was unity, the surfaces near the ridge remained almost fully squashed in even when the mean film thickness was as much as twice the height of the original ridge. In this case, the ridge induces a film thickness variation when entering the contact. The ridge continues to travel at its surface speed, remaining squashed. The film thickness variation travels at the average speed of the contact $u_{avg} = \frac{u_1 + u_2}{2}$. The same behaviour was modelled and predicted by Venner and Lubrecht [69] for an ehl line-contact. This phenomenon may serve in future work as a means of verification for having truly transient behaviour in the CFD approach.

6.2 Fluid Properties

6.2.1 Fluid Density

The density model employed in this study is the one developed by Dowson and Higginson [13], where the fluid density is a non-linear function of pressure, which is used in this study. Fluid density can also be a function of temperature. Wang employed a density model depending on pressure and temperature given by [70]

$$\frac{\rho}{\rho_0} = 1 + \frac{A p}{1 + B p} + D(T - T_0)$$

(6.3)

For a mineral oil the constants are $A = 0.6 \cdot 10^{-9} m^2/N$, $B = 1.7 \cdot 10^{-9} m^2/N$ and $D = 0.6 \cdot 10^{-9} - 0.0007/K$. This density-temperature relation becomes significant for large temperature rises. The current approach could be extended with such a model.
6.2.2 Viscosity

The most important viscosity relations (pressure, temperature and shear-rate) for a typical lubricant are implemented and described in section 2.2.6. Bair has undertaken extensive research into the material properties of lubricants in EHL contacts. He developed more accurate models than the one by Johnsson and Tevaarwerk which could be easily applied inside the CFD methodology. [71] [72] [73] [74]

Molecular dynamics simulations has been used by Martini [75] in EHL contacts and she found that the viscosity of thin films under large shear is subject to shear thinning and oscillation with film thickness. She developed a viscosity model that incorporates both effects which could be applied inside the CFD methodology.

6.2.3 Heat Capacity and Thermal Conductivity

Heat capacity and thermal conductivity are currently assumed to be constant in an ehl-contact. However, Larsson has measured both for a number of common lubricating oils [76] in a pressure range from atmospheric to 1.1GPa and at two temperatures, 295 K and 380 K. He found temperature to have only a marginal effect on thermal conductivity, but thermal conductivity was doubled as the pressure was increased from ambient to 1 GPa. The heat capacity per unit volume was influenced both by the pressure and temperature. Larsson developed two empirical equations based on his findings. These relations should be included in a future thermal analysis of an ehl-contact.

6.3 Finite Volume Deflection

Instead of relying on the more than 120 years old Hertzian contact theory, it would be possible to apply the finite volume methodology to the deflection calculation. A solver like this is already implemented in the OpenFOAM package (Version 1.3) and is called stressedFoam, which is developed for small-strain, linear-elastic deformation of a solid body, with optional thermal diffusion and thermal stresses. This solver could be coupled in a forward iterative manner with the fluid-solution and would provide for realistic thermal boundary conditions. The time-marching nature of stressedFoam caters for a physical under-relaxation which would benefit the stability of the fluid/solid system. If the solid and the fluid meshes align with each other, it would be possible to solve the temperature field of the solid fully implicit with the temperature field of the fluid. A possible mesh structure
Figure 6.1: Finite volume deflection

is shown in figure 6.1. Furthermore this approach could be the basis to model non-homogenous materials such as coating layers and complex material properties such as plastic deformation.

6.4 Deflection / Fluid Coupling

6.4.1 Differential Deflection

Reynolds Approach

The differential deflection method introduced in section 2.3.4 localises the effect of pressure on deflection and promises to increase stability by partially coupling pressure and deflection.

In order to evaluate whether that method is suitable for a CFD approach the procedure for the Reynolds approach is introduced as outlined by Hughes [39]. The one-dimensional steady-state Reynolds equation is

$$\frac{\partial}{\partial x} \left[ \frac{\rho h^3}{\eta} \frac{\partial p}{\partial x} \right] = 12 \frac{\partial}{\partial x} [\rho h U]$$

(6.4)

This equation can be discretised into a system of linear algebraic equations leading to

$$[R_p] \{p\} = \{R_r\} - [R_h] \{h\}$$

(6.5)

Square brackets indicate second rank tensors and curly brackets indicate first rank tensors. $[R_p]$ and $[R_h]$ are banded matrixes. The elastic deflection equation 2.5 can
be expressed as
\[ w_i = \sum_{\text{all}k} g_{k-i} p_k \]  
(6.6)

and the geometry equation 2.3 can be expressed as
\[ h_i = \hat{h}_i + w_i \]  
(6.7)

with \( \hat{h}_i \) describing the undeformed geometry. In tensor notation this becomes
\[ \{h\} = \{\hat{h}\} - [E_p]\{p\} \]  
(6.8)

where \([E_p]\) is a full matrix.

Equation 6.5 and equation 6.8 can be combined in the form
\[ \begin{bmatrix} R_p & R_h \\ E_p & I \end{bmatrix} \begin{bmatrix} p \\ h \end{bmatrix} = \begin{bmatrix} R_r \\ \hat{h} \end{bmatrix} \]  
(6.9)

The submatrices \([R_p]\) and \([R_h]\) are banded, \([E_p]\) is a full matrix and \([I]\) is the unity matrix. With the help of the differential deflection equation 2.40
\[ \nabla^2 w_i = \sum_{\text{all}k} f_{k-i} p_k \]  
(6.10)

the full matrix \([E_p]\) can be converted into a banded matrix. This allows one to employ fast and efficient matrix solvers developed for banded matrices with time savings of up to three orders of magnitude compared to fully coupled methods.

**CFD Approach**

In a CFD approach there is no shared variable, like the film thickness \(h\), between fluid-flow and film-thickness calculation.

It would be desirable to couple the pressure equation 3.68 with the film-thickness equation 2.3 in an implicit manner. As there is no direct film-thickness variable \(h\) in the pressure equation one need to assess where changes in film-thickness has an influence while solving for pressure. Changes in geometry influence the pressure equation by changing the cell-volumes, the surface normals and surface areas of the control volumes. The discretisation of the pressure equation 3.68 for the isobaric cavitation model is outlined in section 3.3.

In the time derivative, changes in surface geometry result in changes in cell-
Changes in cell-volume $V$ and face area vector $S_f$ are therefore only a function of two surface points for a two-dimensional case.

In principle the same route as for the Reynolds approach would be open for the CFD approach. There is a chain of links between the pressure equation 3.68 and geometry equation 2.3 via the cell-volume $V$ and face area vector $S_f$ and 6.11. A novel solver might be capable of combining these equations in a stable and efficient way.

### 6.4.2 Multigrid Methods

The Multilevel/Multi-Integration method developed by Venner [30] enhanced stability and decreased computational time. In principle the same strategy could be applied to a CFD approach. Multigrid methods can be applied to systems of equations and have been used for three-dimensional compressible Navier-Stokes equations in the aerodynamic industry [36]. That could be extended with the Multi-Integration method by Venner.

### 6.4.3 Implicit Deflection/Fluid Coupling

For stability reasons is is desirable to couple the deforming solid with the fluid implicitly. Greenshields and Weller [77] managed to transform the constitutive model for a Hookean solid into a form similar to a momentum equation for a fluid, which allows one to have a combined momentum equation for the fluid/solid system. They applied this method to wave propagation in highly flexible tubes, in which a predicted wave falls within 2% of an approximate analytical solution.

The same method could be applied to an ehl-problem. Remaining problems with stability and/or calculation time could be tackled with above-mentioned Multigrid methods.
6.4.4 Conclusion

In the author’s opinion the most promising route for reaching realistic pressure levels with reasonable computational time is first to implement and validate the finite volume deflection discussed in section 6.3. Then, to implement the implicit deflection/fluid coupling discussed in section 6.4.3. If this is not sufficient to reach desired pressure levels, a multigrid method, discussed in section 6.4.2, could be implemented on top of that, making this approach more stable and faster.

6.5 Cavitation

The cavitation models used in this study suffer from two major constraints. The simulated bearing can only be simulated fully flooded, which is in reality more an exception than the rule. Secondly, they cannot resolve the actual flow structure in the cavitation region, as they assume a homogenous mixture of two fluids.

One approach for interface capturing for multiphase system is the VOF (Volume of Fluid) method. The basic idea is that a multiphase system can be represented as a mixture of phases, where the phase distribution includes sharp transitions between phases. The VOF method is well established for incompressible flows. A complete discussion can be found in the PhD thesis by Ubbink [78]. Weller [79] extended that methodology to compressible flows, which is essential for ehl-problems, and applied it to a nozzle where cavitating diesel is injected into air. In that system one phase is essentially modelled by the Weller cavitation model described in section 3.7 and the other phase is ideal gas. Air and vapour are immiscible. With such an approach, partly flooded bearings could be modelled.

In order to capture the flow structure in the cavitating region, it would be possible to have a VOF-system of compressible liquid and ideal gas, where a source-term for phase-change is introduced.

6.6 3D EHL

6.6.1 Mesh Quality

In section 5.6 three-dimensional results for an ehl-contact were shown. At the moment these calculations are very expensive and relatively unstable. Looking at the computational grid introduced in section 4.4 two shortcomings are apparent.
Firstly, there are too many cells outside the contact area where they are not needed. Secondly, there are big jumps in cell-size, which are detrimental for accuracy and stability. The entire mesh is block-structured consisting only of hexahedrons. These two shortcomings could be addressed by employing embedded refinement where the majority of cells are concentrated near the contact region.

6.6.2 Solution Procedure

In the current approach the entire fluid-domain and solid-domain is solved for every time-step. The timestep is generally in the order of $10^{-8}$ seconds or smaller for stability reasons. That kind of resolution is not necessary for the entire domain. It would be possible to partition the solution domain in two or more sub-domains and solve the equations only in the centre every time-step. Such an approach could reduce the computational time considerably.
Chapter 7

Summary and Conclusions

7.1 Summary

The Reynolds theory for fluid flows, together with the Hertzian contact theory for modelling EHL contacts were presented and the main numerical methods solving these were discussed. Appropriate models for the fluid properties were introduced where the viscosity is piezo-viscous, shear-thinning and temperature dependent and where the density is a function of pressure.

The Navier-Stokes equations and the finite volume method for discretising them were outlined. Two homogenous equilibrium cavitation models were described and compared. The isentropic cavitation model, developed for non-viscous flow, requires unphysical values for the sonic velocity of vapour and for the vapour density. It predicts negative pressure in the cavitation region which is possible. However, the validity of these results is uncertain as it was developed for non-viscous flows and in EHL contact fluid flow is dominated by viscous forces. The isobaric cavitation model is stable for physical values of vapour and maintains the specified cavitation pressure inside the cavitating region. For this model an energy equation was implemented which considers the effects of heat conduction and convection, viscous heating and the heat of evaporation.

An algorithm was developed which implements the Hertzian contact theory within the finite volume framework for two and three dimensions. This algorithm was parallelised and validated against analytical solutions.

The cavitation models and the Hertzian contact theory were coupled together in a forward iterative manner and this method was applied to a series of test cases. Isothermal results were compared to the Reynolds theory and were found to be in
very good agreement and pressure was found to be close to constant across the film thickness in most conditions. Only in rolling conditions with high viscosity and high velocity could small deviations to the Reynolds together with small pressure variations across the film thickness be observed. In thin film, sliding conditions the viscosity varies slightly across the film-thickness due to a complex distribution of shear-rate causing shear-thinning. In thick film, sliding conditions the viscosity varies strongly across the film thickness, caused by shear-thinning.

Thermal calculation were performed for the base viscosities of $\eta_0 = [0.01, 1, 0] \text{ Pa s}$, surface velocity $u = 5.0 \text{ m/s}$ and slide-to-roll ratios of $SRR = [0, 1, 2]$. The thermal, low viscosity results show that, in sliding conditions, a temperature-induced shear band develops which is closer to the slower moving and thus hotter surface. In rolling conditions the temperature increase is not very significant.

The thermal, high viscosity results show significant heating, even in pure rolling conditions. Friction force predictions are very much affected by thermal effects, with differences up to $-88.5\%$ compared to the isothermal condition. In the case of sliding and high velocity, a pronounced temperature-induced shear band evolves close towards the hotter surface. Compared to the isothermal case, the shape of the pressure distributions are very different and have steeper gradients. The thermal, low viscosity cases show no significant pressure variation across the film-thickness. The thermal, high viscosity cases show small pressure variations across the film thickness in the case of rolling and large pressure variations in the case of sliding. Differences from the Reynolds theory were analysed and it was found that terms not modelled by the Reynolds theory assume importance for thick films in isothermal rolling and thermal conditions.

Preliminary results for a three dimensional test case were presented which serve as proof-of-concept for realising three-dimensional calculations of an EHL contact.

Recommendations were given for future development. It would be desirable to overcome stability constraints to reach higher loads. For this it was suggested to model the fluid and the solid using the finite volume method and employ a method which gives fully implicit coupling of both domains. In order to reduce computational time and further increase stability, the use of a multigrid method was recommended. Only steady-state calculation were performed in this study, it would be interesting to perform transient calculation with varying loads, varying surface velocities or rough surfaces. Another aim is to include an interface tracking multiphase system of air/oil in order to model partially flooded and starved contacts. The three dimensional method presented is at a very early stage and it
would be fairly straight forward to reduce the computational effort and increase the resolution in the contact area.

7.2 Conclusions

In this work a CFD approach for elastohydrodynamic lubrication (EHL) was developed using the freely-available package OpenFOAM. This method was applied to isothermal and thermal test cases under moderate loads and steady-state conditions. Very good agreement was found with the Reynolds theory in most cases. For isothermal, high viscosity, high velocity and rolling conditions there were small differences from the Reynolds theory. All thermal calculations under sliding conditions developed a temperature induced shear-band which is closer towards the slower and thus hotter surface. The thermal, high viscosity calculations under sliding conditions showed significant pressure variation across the film thickness due to very large viscosity gradients. The impact of temperature on the friction force is very significant.

The maximum reachable pressure level is limited by numerical stability. For the viscosity $\eta_0 = 0.01 \, Pa \, s$, pressures up to $0.8 GPa$ could be reached. For the viscosity $\eta_0 = 1.0 \, Pa \, s$, pressures up to $1.5 \, GPa$ could be reached. Generally, EHL calculation are more stable with thicker fluid films, lower timesteps, more shear-thinning, higher grid resolution and higher Young’s modulus. For a significant improvement in stability it was suggested to couple fluid and solid fully implicit using the finite volume method for both domains and additionally implement a multigrid method.

Three dimensional results were presented which demonstrates the feasibility of such calculations. Further improvements in calculation time should be fairly straight forward.

Overall, a CFD approach for elastohydrodynamic lubrication has been established and it has been shown that the developed method is capable of giving new insights into the physics of elastohydrodynamic lubrication, especially in cases where the usual assumptions of the Reynolds theory break down.
Bibliography


