Synthesis of artificial turbulent fields with prescribed second order statistics

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Outline

1. Motivation

2. Inflow Generator for LES and DNS

3. Numerical Results

4. Conclusion
1. Motivation

Generation of turbulent fields for Inflow conditions with preferably many prescribed statistical properties (autocorrelation functions, length scales)

**INPUT:**
- **RANS:** integral values, energy, integral lengths
- **LES and DNS:** turbulent fields with fluctuations

**OUTPUT:**
- **RANS:** rough information, time averaged values
- **LES and DNS:** detailed information, time and spatial resolved fluctuations
1. Motivation

Different methods for generating inflow conditions

a) Natural generation of turbulence

b) Use of periodic b.c

c) coupling with LES solutions of auxiliary problems

d) Generation of synthetic turbulent fields
1. Motivation

Overview of statistical methods of different complexity

Level of statistical modelling

1. 1-point space and time statistics
   + (Reynolds stress, kinetic energy)

2. 1-point space and time statistics
   + integral time and spatial length

3. 1-point space and time statistics
   + 2-point space and time autocorrelations

Digital filter based method by (Klein et al. (2001))
Spectral method (Lee et al (1992))

Method of turbulent spots (LTT Rostock)

Modified method of Kraichnan (Smirnov et al (2001), Batten et al (2004))
Diffusion approach (Kempf et al. (2005))
2D random vortex method (FLUENT)........

2D random vortex method (Benhamadouche et al. (2003))
Simple Random Generator (Lund et. al.(1998))......
## 1. Programming

1. Programming is relative complex in 3D, because complicated symmetry conditions have to be fullfilled
2. The grid has to be Cartesian and equidistant
3. Randomness in wave number space

## 2. The grid has to be Cartesian and equidistant

1. Exact solution in the homogeneous case
   - 1. Programming is relative complex in 3D, because complicated symmetry conditions have to be fullfilled
   - 2. The grid has to be Cartesian and equidistant
   - 3. Randomness in wave number space
   - Requires equidistant grid spacing
   - The generation with prescribed autocorrelation function needs the numerical solution (Newton method) even for homogeneous case

## Digital filter based method

1. Easy to code
2. Easy to account for the anisotropy
3. Formulated in physical space

## 3. Randomness in wave number space

1. Exact solution in the homogeneous case
2. Requires equidistant grid spacing
3. The generation with prescribed autocorrelation function needs the numerical solution (Newton method) even for homogeneous case

## Turbulent spots method

1. Grid-independent (can be used for unstructured grid)
2. Analytical solutions for homogeneous turbulence
3. Easy to code
4. Easy to account for the anisotropy
5. Formulated in physical space

## Comparison of methods of 3rd level

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
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</thead>
<tbody>
<tr>
<td>Spectral method</td>
<td>Exact solution in the homogeneous case, 1. Easy to code</td>
</tr>
<tr>
<td></td>
<td>2. Easy to account for the anisotropy, 3. Formulated in physical space</td>
</tr>
<tr>
<td>Digital filter based method</td>
<td>1. Requires equidistant grid spacing, 2. The generation with prescribed</td>
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<tr>
<td></td>
<td>autocorrelation function needs the numerical solution (Newton method)</td>
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<td></td>
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<tr>
<td>Turbulent spots method</td>
<td>1. Grid-independent (can be used for unstructured grid), 2. Analytical</td>
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<tr>
<td></td>
<td>solutions for homogeneous turbulence, 3. Easy to code</td>
</tr>
<tr>
<td></td>
<td>4. Easy to account for the anisotropy, 5. Formulated in physical space</td>
</tr>
</tbody>
</table>

## Disadvantages

- Easy to code
- Easy to account for the anisotropy
- Formulated in physical space
1. Motivation

2. Inflow Generator for LES and DNS
   2.1 Synthesis of non divergence free turbulent fields
   2.2 Synthesis of divergence free turbulent fields
   2.3 Calculation scheme

3. Numerical Results

4. Resume
2. Inflow Generator for LES and DNS

Fluctuations

\[ u^{(n)}(\vec{x}) = \sum_{i=1}^{M} r^{(n)}_i \phi_i (\vec{x} - \vec{x}^{(n)}_{ri}) \]

2.1 Synthesis of non divergence free Fields

\[ \rho(\tilde{\eta}) = \frac{u(\tilde{x})u(\tilde{x} + \tilde{\eta})}{u(\tilde{x})} \]

The two-point correlation and the energy spectrum function are given by

\[ R(\tilde{\eta}) = u(\tilde{x})u(\tilde{x} + \tilde{\eta}) = C^2 \int_{-\infty}^{\infty} \phi(\eta) \rho(\eta) d\eta \]

Exact solution for homogeneous turbulence:

\[ \phi(\tilde{x}) = C_{\Sigma} F_1^{-1} \{ F_1^{1/2} \{ \rho(\eta,0,0) \} \} \cdot \rho(\tilde{x},0) \]

for inhomogeneous turbulence - numerical solution:

\[ \int_{\eta - \rho(y_s + \eta)}^{\eta + \rho(y_s)} \int f(y_s, y, \rho(y_s)) f(y_s + \eta, y, \rho(y_s + \eta)) dy - R(y_s, \eta) \int_{y_s - \rho(y_s)}^{y_s + \rho(y_s)} f^2(y_s, y, \rho(y_s)) dy = 0 \]

2.2 Synthesis of divergence free homogeneous fields

\[ \vec{u} = \nabla \times \vec{A} \]

\[ \nabla \vec{u} = \nabla (\nabla \times \vec{A}) \equiv 0 \]

\[ \nabla \vec{\omega} = \nabla (\nabla \times \vec{u}) \equiv 0 \]

\[ \vec{A} = A(\rho_1, \rho_2, \rho_3)\hat{e}_3 \delta \]

\[ R^A_{ij} = \langle A_i(\vec{r})A_j(\vec{r} + \vec{\eta}) \rangle = C \int_0^\pi \int_0^{2\pi} \int_0^{2\pi} A(\vec{r})A(\vec{r} - \vec{\eta})E_{3i}E_{3j}d\vec{r}P(\alpha, \beta, \gamma) \sin \alpha d\alpha d\beta d\gamma \]

Simple case: spherical carrier of vector potential

\[ A(\vec{r}) = \frac{1}{2\sqrt{\pi}C^{1/2}} \int_0^\infty \int_0^{2\pi} \frac{E(k)}{k^2} \cos(kr \cos \varphi)k^2 \sin \varphi dk d\varphi \theta = \frac{2\sqrt{\pi}}{C^{1/2}} \int_0^\infty \sqrt{E(k)} \frac{\sin kr}{kr} dk \]

N.Kornev, E.Hassel (2007), Physics of Fluids, accepted.
### Structures

<table>
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<tr>
<th>Vorton chain</th>
<th>Filament</th>
<th>Scalar sphere</th>
</tr>
</thead>
</table>

#### Solution

**Isotropic case:**

\[ \mathbf{A}(r) = F^{-1}(B(k)) \]

**Anisotropic case:**

\[ \mathbf{A}(r) = 2\sqrt{\pi / C} \int_0^\infty \sqrt{\frac{E(k) / K}{D_{11}(k) + \frac{1}{4} D_{33}(k) / kr}} \sin kr dk \]

\[ D_{ij} = \int_0^{2\pi} \int_0^\pi E_{3i} E_{3j} P(\alpha, \beta) \sin \alpha d\alpha d\beta + \]

\[ + 2 \sum_{m=1}^{K-m} \frac{K-m}{K} \int_0^{2\pi} \int_0^\pi E_{3i} E_{3j} \cos[\varepsilon km \cos \alpha] P(\alpha, \beta) \sin \alpha d\alpha d\beta \]

**Scalar sphere**

\[ S(r) = 2\sqrt{2\pi / Cr} \int_0^\infty \sqrt{E_S(k)} \sin kr dk \]
2.3 Calculation scheme

Autocorrelation functions
\[ \rho_{uu}(\eta,0,0), \rho_{uu}(0,\eta,0), \rho_{uu}(0,0,\eta) \]
\[ \rho_{vv}(\eta,0,0), \rho_{vv}(0,\eta,0), \rho_{vv}(0,0,\eta) \]
\[ \rho_{ww}(\eta,0,0), \rho_{ww}(0,\eta,0), \rho_{ww}(0,0,\eta) \]

Reynolds Stresses
\[
\begin{pmatrix}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33}
\end{pmatrix}
\]

Integral Length
\[ L = \int R(x, \Delta x) \, dx \]

predefined Autocorrelation

Inflow Generator

Stochastic signal with statistics prescribed in INPUT

Experiment, auxiliary LES

LES Field
1. Motivation

2. Inflow Generator for LES and DNS

3. Numerical Results
   3.1. Influence Parameters
   3.2. Coaxial Jet Mixer
   3.3. Swirl Injector

4. Resume
3. Numerical Results

Case: Free Decaying Turbulence

Autocorrelation functions
\[ \rho_{ww}(\eta,0,0), \rho_{ww}(0,\eta,0), \rho_{ww}(0,0,\eta) \]
\[ \rho_{ww}(\eta,0,0), \rho_{ww}(0,\eta,0), \rho_{ww}(0,0,\eta) \]
\[ \rho_{ww}(\eta,0,0), \rho_{ww}(0,\eta,0), \rho_{ww}(0,0,\eta) \]

Inflow Generator

Inlet oscillations
3.1 Influence Parameters

- intermittency
- autocorrelation function
- integral length
- divergence of the inflow velocity
3.1 Influence Parameters

autocorrelation function

\[
\rho(\eta) = \begin{cases} 
\frac{3}{4} (\eta/L)^2 - \frac{3}{2} (\eta/L)^3 + 1, & \text{if } 0 \leq \eta \leq L \\
2 - \frac{3}{2} (\eta/L) + 3/2 (\eta/L)^2 - 1/4 (\eta/L)^3, & \text{if } L \leq \eta \leq 2L 
\end{cases}
\]

Hat:

Qube:
3.1 Influence Parameters

integral length

\[ \nabla \cdot \vec{U} = 0 \]
3.1 Influence Parameters

divergence $\nabla \cdot \vec{U} = 0$
3.1 Influence Parameters

intermittency

divergence free inlet conditions

Kinetic turbulent energy

x
3.2 Coaxial Jet Mixer

D – diameter pipe (50 mm)

\(d\) – diameter nozzle (10 mm)

\(U_d\) – bulk velocity jet (1.23 m/s)

\(U_D\) – bulk velocity coflow (0.069 m/s)

\(Re_d = U_d d / \nu = 10.000\)

\(x/D = 1.6\)
3.2 Coaxial Jet Mixer

- Inflow Generator weak
- Random fluctuations strong
- Convergency with Inflow Generator when resolution increases
- No delay of mixing in axial length

Inflow Generator is necessary
3.3 Swirl Injector

Vortex structures of the flow inside a swirling jet

(isosurface at $\lambda_2 = -5 \times 10^4$)

Random

Inflow Generator
4. Conclusions

Turbulent spots method as a new efficient technology for synthesis of turbulent fields with prescribed second-order statistics

- is successfully implemented in OpenFOAM
- generates unsteady inlet conditions for LES and DNS
- has physical clear background
- is universal to the inlet grid

- following problems have been solved
  - Generation of homogeneous and inhomogeneous anisotropic turbulent fields
  - Generation of divergence free homogeneous anisotropic vortex fields
- Experience with free decaying turbulence
  - integral length
  - autocorrelation functions
  - intermittency
  - divergence free signal could shorten the transition range

strong influence
4. Resume

Further Investigations

- Investigations of Inflow Generator for turbulent channel flows
- Development of a new SGS-model for the mixing of fluids at high Schmidt numbers

Scalar structures in a coaxial jetmixer
spatial resolution : 31 μm
Thank you for your attention

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